MATHEMATICS ENRICHMENT CLUB. Problem Sheet 17, September 17, 2018

1. Suppose that

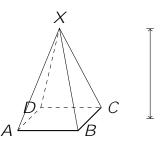
 $N = 1^9 \quad 2^8 \quad 3^7 \quad 4^6 \quad 5^5 \quad 6^4 \quad 7^3 \quad 8^2 \quad 9^1$:

How many perfect squares divide N?

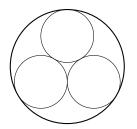
2. Let 10 a; b; c 10. How many triplets, (a; b; c), satisfy

$$\frac{a=b}{c} = \frac{a}{b=c}?$$

3. A right square-based pyramid is placed on a table. The pyramid has base *ABCD* with sides of length *b* and apex *X* at a height *h* above the base. What is the shortest distance an ant on the table can travel when moving from *A* to *C*?



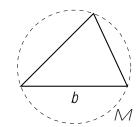
4. Three small circles with radius *r* are inscribed in a larger circle with radius *R* as shown in the diagram.



What is the relationship between r and R?

5. Contruction Problem

(a) Show that you can construct the circumcircle, M, of a triangle given the length of the base, *b*, and the angle at the apex, .



(b) Construct a triangle, given the angle at the vertex; the length of the altitude from the vertex to the base; and the length of the median from the vertex to the base.

Senior Questions

- 1. A napkin ring is formed by drilling a hole of length *h* through the centre of a sphere of radius *r*. Find the volume of the napkin ring.
- 2. (a) Show that, for 1 < x < 1

$$\frac{1}{1+x} = 1 \quad x + x^2 \quad x^3 + \dots;$$

and hence show that

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

(b) Using the result from part (a), how many terms are needed to approximate ln(1:1) correct to 5 decimal places?