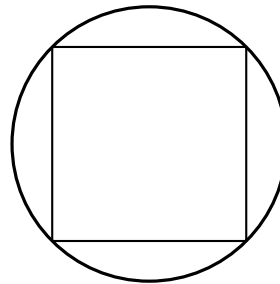
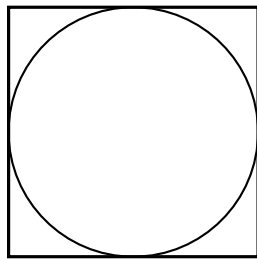


**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 12, August 13, 2018**

1. Two pizza places differ in how they prefer to box their pizzas. One produces circular pizzas and delivers them in a square box, while the other produces square pizzas and delivers them in a circular box.



Who is wasting a higher proportion of their box space?

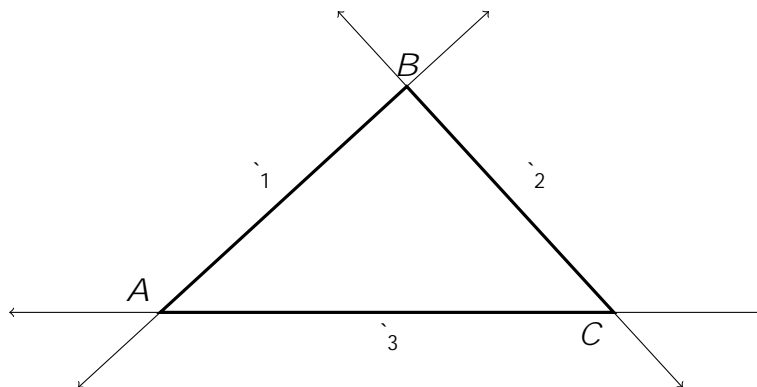
2. When multiplying two whole numbers a student by mistake reduced the tens digit in the answer by 7. She checked her answer by dividing it by the smaller factor, obtaining the quotient 48 and the remainder 17. Find the two factors.
3. Find all integers  $x$ ,  $y$ , and  $z$  such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{x+y+z};$$

4. The number  $N$  is less than 500 and has three distinct digits, none of them 0. Five different numbers can be obtained by changing the order of the digits of  $N$ . The arithmetic mean of these five numbers is equal to  $N$ . Find  $N$ .

*Questions continue overleaf.*

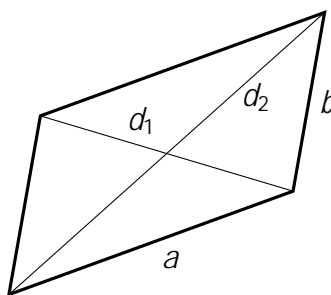
5. Let  $\ell_1$ ,  $\ell_2$  and  $\ell_3$  be three lines that intersect pair-wise at  $A$ ,  $B$  and  $C$ . (Thus  $A$ ,  $B$  and  $C$  form a triangle.) Find all points in the plane that are equidistant from  $\ell_1$ ,  $\ell_2$  and  $\ell_3$ <sup>1</sup>.



6. We only know that the password of a safe consists of 7 different digits. The safe will open if we enter 7 different digits, and at least one of them matches the corresponding digit of the password. Can we open this safe in less than 7 attempts?

### Senior Questions

1. (a) **The Parallelogram Law.** The diagram below shows a parallelogram with sides  $a$  and  $b$ , and diagonals  $d_1$  and  $d_2$ .



Use the cosine rule to show that

$$d_1^2 + d_2^2 = 2(a^2 + b^2):$$

- (b) Now prove the parallelogram law using complex numbers. That is, show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2):$$

*Hint: first show that  $jz^2 = \bar{z}z$ .*

<sup>1</sup>Adapted from AP Kiselev *Kiselev's Geometry: Planimetry*, Tr. A Givental, 2006