[redacted]

$ on drinks.

3. Line up 40 lots of the number "1", that is

$$\underbrace{111\ldots111}_{40\text{ lots}}$$

Then we can think of this problem as partitioning the list of number 1's into 4 blocks (allowing some block(s) to be empty); The sum of the numbers in each block will be the value of either x_1, x_2, x_3 or x_4, and the sum of all blocks will always be 40 because we have a list of 40 lots of "1", i.e $x_1 + x_2 + x_3 + x_4 = 40$.

To find the number of ways we can partition the 40 objects into 4 blocks, one can think of it as the number of ways to arrange 43 objects (with 3 walls that separates the 40 objects into blocks). The total number of ways we can do this is 43!, but we don't care about how the objects are arrange because they are all 1's, nor do we care about how the walls are arranged. Hence, the number of solutions is 43!   40!   3! = 12341.

4. This is a straightforward application of integration by parts `https://en.wikipedia.org/wiki/Integration_by_parts`. The solution is 5! = 120.

5. Let $r_1; r_2$ be the remainder when the integers $x_1; x_2$ is divided by some integer d respectively. One can show that the remainder of $x_1 + x_2$ divided by d is $r_1 + r_2$, and the remainder of $x_1 \quad x_2$ is $r_1 \quad r_2$ (write $x_1; x_2$ in quotient and remainder form, then computer $x_1 + x_2 / \; x_1 \quad x_2$).

   The remainder of 1^n divided by 7 is aways 1.

   The remainder of 2^1 divided by 7 is 2. Hence, the remainder of $2^2 = 2 \quad 2$ divided by 7 is $2 \quad 2 = 4$. Similarly, the remainder of $2^3 = 2^2 \quad 2$ divided by 7 is $4 \quad 2 = 8$, which is the same as 1. Finally, the remainder of $2^4 = 2^3 \quad 2$ divided by 7 is $1 \quad 2 = 2$. Since the remainder of 2^4 divided by 7 is the same as 2^1, if we keep increasing the power n in 2^n we would expect the remainders to be $2; 4; 1$ repeating, because we are multiplying the remainder by the same factor of 2 each time.

   Analogously, the remainders of 4^n divided by 7 are $4; 2; 1$ repeating.

   Therefore, the remainder of $1^n + 2^n + 4^n$ divided by 7 is $1 + 2 + 4; 1 + 4 + 2; 1 + 1 + 1$ repeating, which is the same as $0; 0; 3$ repeating. Thus, we can argue that $2{=}3$ of 999 the numbers between $0 < n < 1000$ makes $1^n + 2^n + 4^n$ divisible by 7.

6. Draw the line BO, where O is the origin of the circle; see below. From the gure, we can easily nd x using Pythagoras on the 2 right angled triangles. The length of BC is twice that of $x \quad 2$.