

MATHEMATICS ENRICHMENT CLUB.  
Problem Sheet 4, May 22, 2016

1. Find

$$S = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots$$

### Senior Questions

1. Prove that the equation  $x^7 + y^9 = z^8$  has infinitely many solutions in positive integers  $x; y$  and  $z$ , all powers of 2.
2. Every term of an infinite geometric progression is also a term of a given infinite arithmetic progression. Prove that the common ratio of the geometric progression is an integer.
3. The incircle of the quadrilateral  $ABCD$  touches  $AB; BC; CD$  and  $DA$  at  $E; F; G$  and  $H$  respectively; see below
  - (a) Recall that the *incentre* of a triangle is the point where the internal angle bisectors of the triangle intersect. Show that the incentre of  $AEH$  lies on the incircle of  $ABCD$ .
  - (b) Show that the incentres of triangles  $HAE$  and  $FCG$  is perpendicular to the line joining the incentres of triangles  $EBF$  and  $GDH$ .

