

MATHEMATICS ENRICHMENT CLUB.¹ Problem Sheet 7, June 9, 2012

- 1. In how many ways can 6 boys and 6 girls stand next to each other in a row such that no two boys stand next to each other and no two girls stand next to each other?
- 2. The notation 5! means $5 \times 4 \times 3 \times 2 \times 1 (= 120)$. How many zeros are there at the end of 1000!.
- 3. (a) a;b are positive numbers with a+b=k. Explain why ab is greatest when $a=b=\frac{k}{2}$:
 - (b) Suppose that $x^2 + y^2 = c^2$, and the minimum value of $x^4 + y^4$.
- 4. (a) Show that there are in nitely many non-zero integers x; y; z such that $2^x + 2^y = 2^z$:
 - (b) Show that if n > 2 then there are no nonzero integers x; y; z such that $n^x + n^y = n^z$:
- 5. (Parts b and c require Year 9 and Year 10 Mathematics). Let ABC be an isosceles triangle with the base angles B and C being 72° and AB = AC = 4. The length of the base BC, called X is chosen such that a line CD can be drawn, where D lies on AB, such that $\angle BDC = 72^\circ$.
 - (a) Find a pair of similar triangles and show that x satis es, $x^2 + 4x 16 = 0$:
 - (b) Use triangle ABC to nd cos 72° in surd form.
 - (c) Use triangle ACD to nd cos 36° in surd form.
- 6. Suppose that two non-parallel straight lines k and `meet at a point P which is **not** on the page of my book. Construct a line which would (if P did lie on the page) bisect the angle between the lines and pass through P.
- 7. Let K; L be points on the sides AB; AD respectively of the convex quadrilateral ABCD such that $AK = \frac{1}{3}AB$ and $AL = \frac{1}{3}AD$: Similarly, M; N are points on CD; CB such that $CM = \frac{1}{3}CD$ and CN

Year 11 Question.

1. Suppose that *m* and *n* are positive real numbers. Use trigonometry to nd the the maximum value of

$$\frac{m+n}{\sqrt{m^2+n^2}}.$$