

MATHEMATICS ENRICHMENT CLUB.¹ Problem Sheet 5, May 28, 2012

- 1. Two classes of 20 and 30 students average 66% and 56% respectively on an examination. What is the average for all the students on the exam?
- 2. A mathematics test has 5 questions on each of which people can score 0,1,2 or 3 marks. How many ways can a student receive a total of 12 marks $\frac{1}{2}A_{12}A_{3}A_{6}A_{9}$?
- 4. Find in nitely many integers x such that

$$\frac{1}{X+p_{\overline{X^2+1}}} + \sqrt[3]{X-p_{\overline{X^2+1}}}$$

is an integer.

- 5. (a) Prove that $a + b = 2^{p} \overline{ab}$ for any positive real numbers a; b.
 - (b) Deduce that for x; y; z positive, (x + y)(x + z)(y + z) = 8xyz:
- 6. In the triangle *ABC*, it is given that $\angle ABC = 140$. Let *D* be a point on *AC* and *E* a point on *AB* such that the three triangles *AED*; *EDB* and *DBC* are all isosceles, with their vertices at *E*; *D* and *B* respectively. Find all the angles of the triangle *ABC*.
- 7. Let *ABCD* be a trapezium and with *ABjjCD*. Let *M*; *N* be the midpoints of *AD* and *BC* respectively. Show that $MN = \frac{1}{2}(AB + CD)$:

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions.

1. Let
$$f(x) = \left(1 + \frac{1}{x}\right)^x$$
.
(a) Prove that $\frac{f^{\theta}(x)}{f(x)} = \log\left(1 + \frac{1}{x}\right) - \frac{1}{1 + x}$:
(b) By considering the area under the curve $y = \frac{1}{t}$ for t from 1 to $1 + \frac{1}{x}$, show that $\log\left(1 + \frac{1}{x}\right) > \frac{1}{1 + x}$ and deduce that $f(x)$ is increasing.

- 2. Suppose a > b > 0. Find $\lim_{n! \to 1} (a^n + b^n)^{\frac{1}{n}}$.
- 3. By considering cos(A + B) + sin(A B) = 0 nd the general solution (for) of cos n + sin m = 0.