MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 2, May 7, 2012

1. Find all pairs of primes $(p;q)$ such that p divides $q^2 \quad q$ and q divides $p^2 + p$.

2. If x is a number between 4 and 8 and y is a number between 20 and 40, what are the smallest and largest possible values of $\frac{y}{x}$?

3. Write the quartic $x^4 + 4$ as the product of two quadratics. What about $x^4 + 1$?

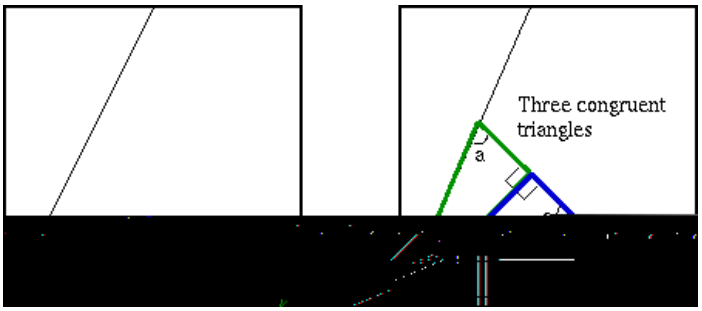
4. Find all positive integers $x;y;z$ such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{8}$:
(Hint: Suppose $x \quad y \quad z$ and hence nd the possible values of x.)

5. Suppose that a set S contains the numbers 1,2,3,4 and that the sum of any four different numbers in S is also a number in S. Show that S contains every positive integer greater than 21.

6. Let ABC be a triangle whose incircle has radius r. Let s equal to half the perimeter of the triangle. Show that the area of the triangle is rs:

7. Let S be the centroid of the triangle ABC and let $A_1;B_1;C_1$ be the midpoints of the sides, (with A_1 opposite A and so on). If AA_1 meets B_1C_1 at G, nd the ratio of the areas of B_1GS to the area of ABC.

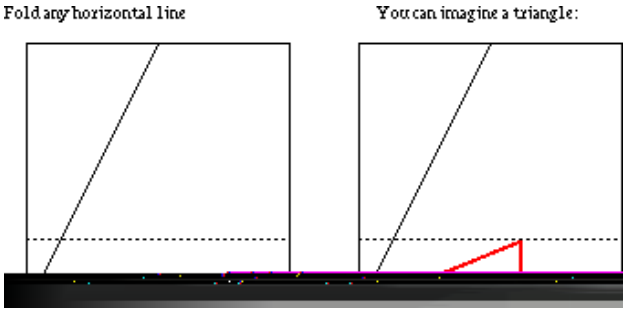
8. Tricsecting the angle (taken from www.math.lsu.edu/ verrill/origami/trisect/)
Suppose we could put three congruent triangles in the picture as shown:



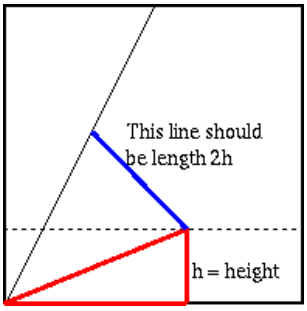
[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

These triangles trisect the angle. So we need to know how to get them there.

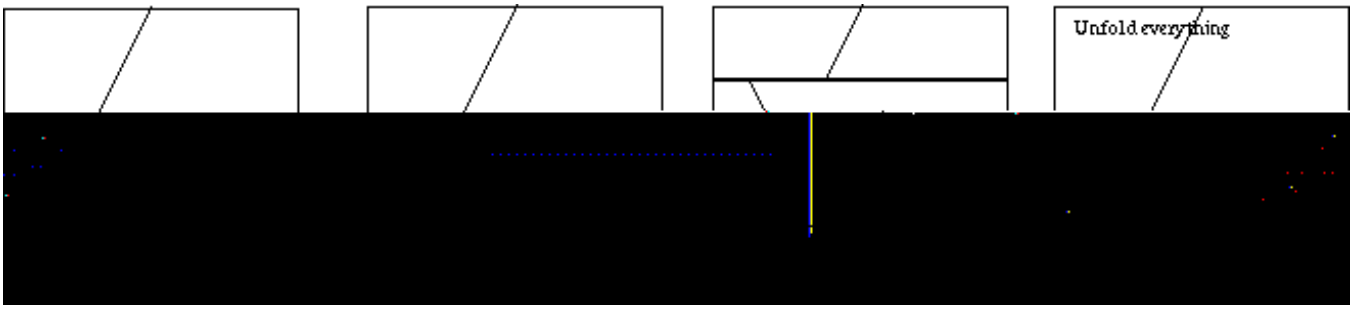
Choose some height for the lower triangle, any height, and crease a horizontal line at this height; ie, just crease any horizontal line you want:



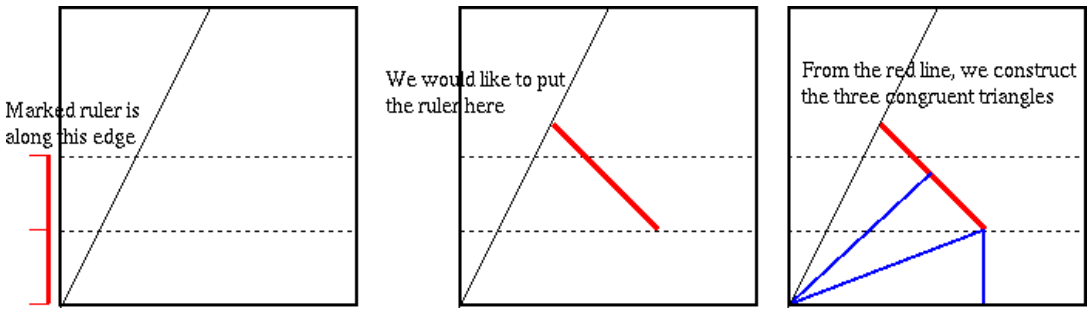
We need to get the blue line of the following picture somehow:



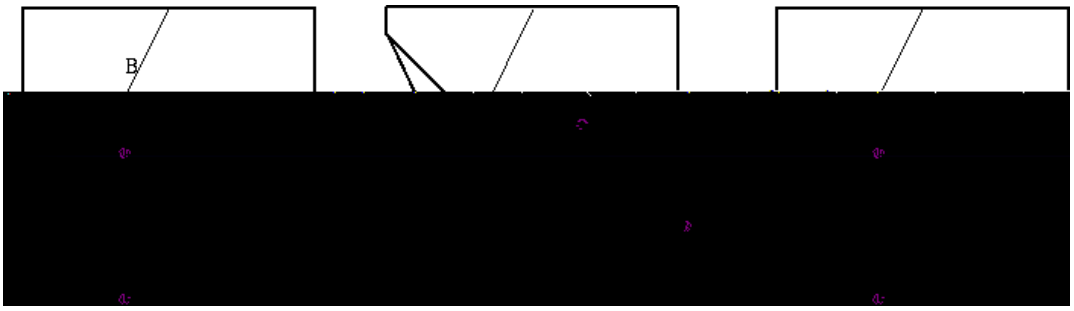
We can make a kind of "marked ruler" in the side of the paper, by folding over the paper again:



Now this "marked ruler" is used to nd the bold line we needed:



To do this, fold the paper so point b touches line B, and point d touches line D:



Senior Questions.

1. If x is positive, how large must x be so that $\sqrt{x^2 + x}$ x di ers from $\frac{1}{2}$ by less than 0:02?

2. Find the largest integer that exactly divides $11^{k+2} + 12^{2k+1}$ for all positive integers k.

3. Solve the equation $\cot^{1} x$ $\cot^{1}(x + 2) = \frac{\pi}{12}$: