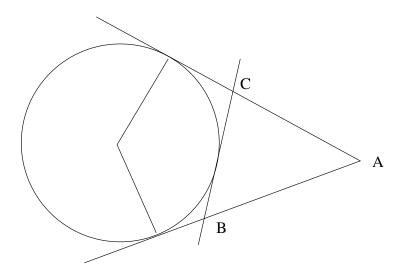


MATHEMATICS ENRICHMENT CLUB.¹ Problem Sheet 16, September 10, 2012

- 1. A box of apples costs \$4, a box of oranges costs \$3 and a box of lemons costs \$2. A person buys 8 boxes of fruit at a cost of \$23. If at least one box of each kind of fruit is bought, find the largest possible number of boxes of apples.
- 2. Suppose a, b are positive real numbers. Use the diagram below to give a **geometric proof** that $a^2 = (a b)^2 + 2ab b^2$.

- (b) Paul then measured the edges to be 2,3,4,5,6,8. If AB=2 what is the length of CD?
- 7. A circle is drawn which touches BC in triangle ABC and also touches the two sides AB and AC produced at T and S respectively. Let O be the centre of this circle.



- (a) Explain why OB bisects the angle TBC.
- (b) Prove that the length of AT equals half the perimeter of the triangle ABC.

0.1 Senior Questions.

1. Prove that

$$cos((n+2)) = 2cos((n+1))cos - cos(n),$$

for each integer $n \ge 0$.

Hence express $\cos 5$ in terms of powers of \cos .

2. For every positive real number n > 1, prove that

$$2\sqrt{n+1} - 2\sqrt{n} < \frac{1}{\sqrt{n}} < 2\sqrt{n} - 2\sqrt{n-1}.$$

3. Use the result in Q2 to prove that

$$2(\sqrt{N+1}-1) < \sum_{n=1}^{N} \frac{1}{\sqrt{n}} < 2\sqrt{N}$$

and deduce that the sum of the first million terms of

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots$$

is between 1998 and 2000.