

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 15, September 3, 2012

1. In how many ways can we change \$10 into 50 cent and 20 cent coins, with at least one of each coin being used.

2. If $x = \frac{q}{1 + \frac{p}{1 + \frac{p}{2}}}$ find the exact value of $x^4 - 2x^2$.

3. A quadrilateral in which a circle can be drawn which touches each of the four faces is called a *circumscribable quadrilateral*. If r is the radius of the circle and s is half the perimeter of the quadrilateral, prove that the area of the quadrilateral is rs .

4. Use the fact that $2xy = (x + y)^2 - x^2 - y^2$ to show that

$$2(b - c)(c - a) + 2(c - a)(a - b) + 2(a - b)(b - c) = 0$$

for all real numbers $a; b; c$.

5. (a) Find all positive integers $a; b; c$ such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is as large as possible but less than $\frac{1}{2}$:

(b)

2. Prove by induction that the sum to k terms of

$$1^2 + 3^2 + 5^2 + 7^2 + \dots$$

equals $8n^2$ when $k = 2n$ and $8n^2 + 8n + 1$ when $k = 2n + 1$.

3. In $\triangle ABC$ prove that $b^2(\cot A + \cot B) = c^2(\cot A + \cot C)$. (Hint: You might begin by considering the area of the triangle in two different ways.)