

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 11, August 2, 2012

1. Solve $\frac{x + 3y}{2x + 5y} = \frac{4}{7}$.
2. Find a number less than 100 which is increased by 20% when the digits are reversed.
3. (a) Verify that

$$\begin{aligned} x^{15} - 1 &= (x^3 - 1)(x^{12} + x^9 + x^6 + x^3 + 1) \\ &= (x^5 - 1)(x^{10} + x^5 + 1): \end{aligned}$$

- (b) Hence factor $2^{15} - 1$ as a product of prime factors.
- (c) Can you factorise $2^{15} + 1$ as a product of prime factors?
4. Suppose that P is a point inside a rectangle $ABCD$ with $AB = 15\text{cm}$, and $AD = 10\text{cm}$. If $PA = 14\text{cm}$ and $PB = 11\text{cm}$, find PD in surd form.
5. Find all positive integers m and n such that $3m - 1$ is a multiple of n and $3n - 1$ is a multiple of m .
 (Hint: Suppose $m = n$, then n divides $3m - 1 < 3m - 3n$.)

6. (a) Let M be the midpoint of the side BC of the triangle ABC and let N be the midpoint of AC . Suppose that AM and BN meet at S . Show that

$$AS : SM = BS : SN = 2 : 1:$$

- (b) Hence show that the medians of a triangle are concurrent.
7. (a) Let M be the midpoint of the side AB in the triangle ABC . If CM has length h , prove that

$$2(a^2 + b^2) = c^2 + 4h^2:$$

This is known as **Apollonius' theorem**.

- (b) Show how to draw a triangle knowing only the lengths of the three medians $h; k$ and l . (You can either use (i), or find a better way.)

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.