Dependence modeling in General Insurance using LGC and HMMs

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Dependence modeling-Motivation

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- Non-linear and tail dependence- an issue?

Why LGC and HMMs?

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HMMs

- Capture complex temporal dependencies
- Handling Varying Patterns and Regimes.
- Assumes that data-generating process corresponds to a time-dependent **mixture** of conditional distributions driven by **Hidden states**

LGC basic notation

Define

- Let $X = (X_1, X_2)$ 2D RV with density $f(x) = f(x_1, x_2)$ and ψ be a Guassian bivariate
- $\mu(x) = (\mu_1(x), \mu_2(x))$ is the local mean vector
- $\Sigma(x) = \sigma_{ij}(x)$ is the local covariance matrix.

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- Approximate f locally at each point x by $\psi_x(v)$
- $v = (v_1, v_2)^T$ is the running variable
- The local population parameters $\lambda(x) = (\mu(x), \Sigma(x))$ can be defined by minimizing a penalty function q measuring the di□erence between f and ψ

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- Used TMB


QUESTION?

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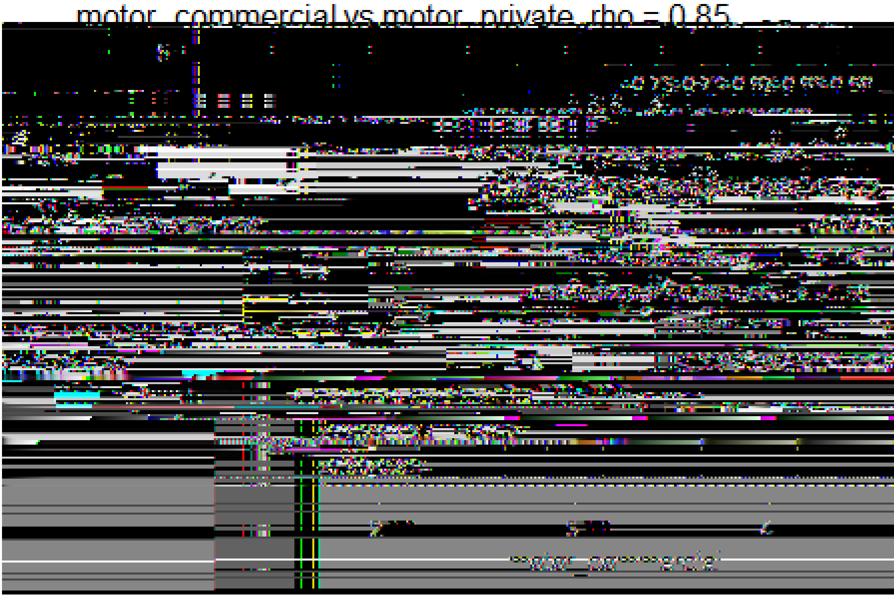
- 7 lines of business considered(Personal Accident, Fire industrial, Motor private, motor comprehensive, Workers compensation,liability, Engineering)
- LGC and HMM Illustrated using Motor LoBs and Homeowners Insurance LoBs
- For motor-15 years from July 2007 to Dec 2021 giving 756 weekly records.


motor_commercial vs motor_private, rho = 0.85

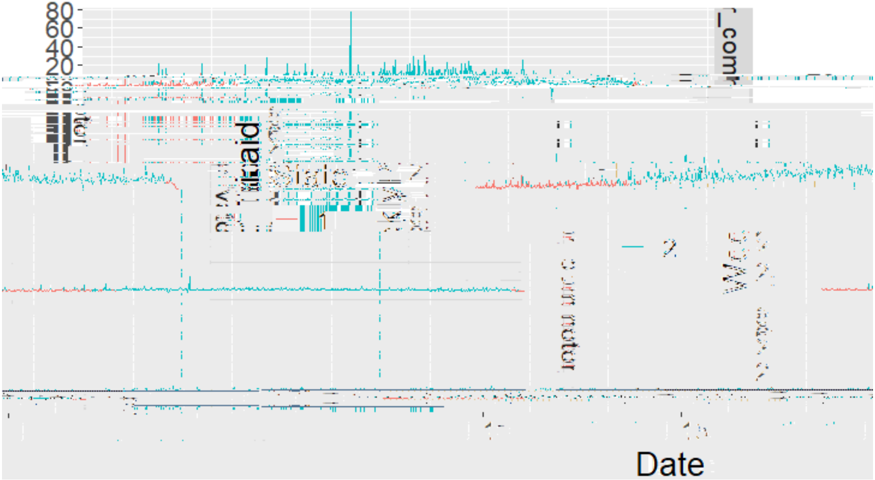
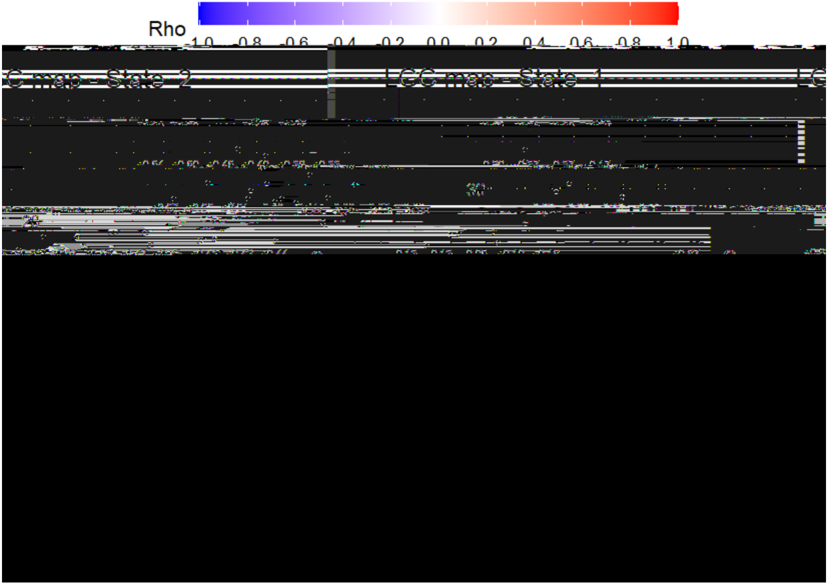
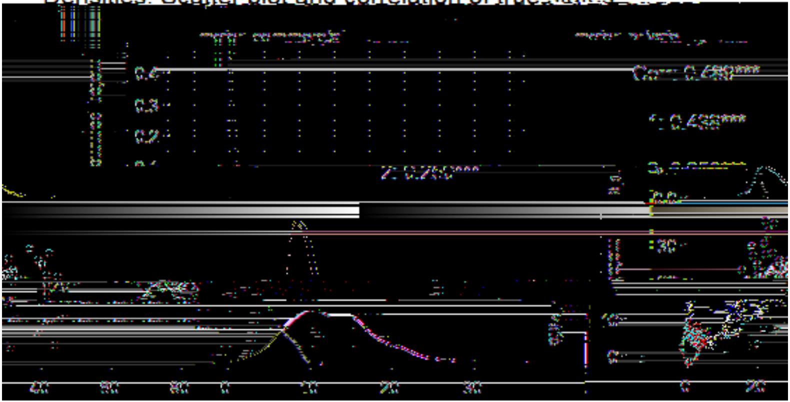

80
60
40
20
Date

Table: Table central measures

Descriptive measures										
LoB	States	n	mean	sd	median	min	max	Skew	Kurtosis	se
Motor-commercial	state 1	615	4.41	3.53	3.74	-1.23	15.789	0.69	-0.33	0.14
	state 2	141	11.65	9.14	10.53	-12.59	77.03	2.62	16.79	0.77
	All states	756	5.76	5.79	4.44	-12.59	77.03	3.42	30.96	0.21
Motor-private	state 1	615	9.09	5.94	8.94	-1.74	27.33	0.3	-0.69	0.24
	state 2	141	15.34	7.11	15.29	-1.04	36.52	0.54	0.27	0.6
	All states	756	10.25	6.64	9.99	-1.74	36.52	0.52	0.2	0.24
Workers-Compen	state 1									


Rho
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
LGC map - State 1

Densities, Scatter plot and correlation of true states



Test of asymmetric dependence between states

Hypothesis:

$$H_0 : \quad \rho_1(x_i, y_j) =$$

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- This analysis enables insurers to better assess and manage their overall risk exposure across different lines of business especially during economic,political crisis periods

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- Compare with other dependency modeling techniques like Bernstein copulas, Vine Copulas?
- Include covariates? claim reporting delays, settlement delays,
- Apply to Reserving, pricing,reinsurance arrangements?