Optimal Savings and Portfolio Choice with Risky Labor Income and Reference-Dependent Preferences

Servaas van Bilsen
Amsterdam School of Economics
University of Amsterdam
Netspar

based on joint work with

Roger Laeven and Theo E. Nijman

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1. Introduction

- How to optimally save and invest total wealth over the life cycle has been studied extensively.
- Human capital constitutes the largest part of total wealth.
  - For average US household, it is 90% of total wealth.
- Impact of risk-free and tradable labor income on optimal choice is well understood.
  - Bond-like human wealth diversi es stock return risk.
- Some authors explore the impact of non-tradable risky labor income but assume traditional preferences.
  - CRRA or Epstein-Zin.
- We explore joint impact of reference-dependent preferences and non-tradable risky labor income on optimal savings and portfolio decisions.

Risky Labor Income and Reference-Dependent Preferences

- Labor income is not risk-less as has been vividly illustrated by the recent Covid-19 crisis.
  - US unemployment rates rise from 3.5% to 14.7%.
- A large experimental and empirical literature has shown substantial deviations from traditional preferences.
  - Reference-dependence is one of the strongest empirical phenomena in decision under risk.
- To understand how risky human wealth a ects optimal savings and portfolio decisions is of great importance.
- This paper analyzes this question for an individual with reference-dependent preferences.

Three Main Findings

2.

Three Main Findings

3. Optimal investment strategy is more conservative compared to the case with risk-less labor income and CRRA preferences.
   - Non-tradable risky labor income causes the optimal share invested in the risky stock to decrease.
   - An endogenous reference level has two additional counteracting e ects on the optimal portfolio share.
   - For a typical range of parameter values, we nd the net e ect yields a reduction in the optimal share invested in the risky stock.

Relating Our Findings to Empirical Analysis

- Our analysis generates several testable implications.
- We brie y explore how our main ndings relate to real monthly savings data.
- Using monthly data on total expenditures and incomes, we test the excess sensitivity of the optimal savings rate and the heterogeneous response of optimal savings rate.
- Consistent with our main ndings, we nd excess sensitivity of the optimal savings rate; and nd that the optimal savings rate of a low-income individual exhibits higher degree of excess sensitivity than that of a high-income individual.

2. Model

Preferences:

- Denote by $c(t)$ and $h(t)$ the individual's consumption level and reference level at time t.
- Expected lifetime utility is given by

$$U = E_0 \int_0^{T_D} e^{-\rho t} u(c(t) - h(t))\, dt;$$

  with $\rho > 0$ time preference rate and T_D the date of death.
- We impose weak assumptions on the utility function u.
- In the base model, we assume that the reference level satisfies

$$dh(t) = (c(t) - \delta(t) h(t))\, dt;$$

  where the depreciation rate δ is allowed to be time-dependent.
- In a more general specification, the reference level is allowed to depend not only on own past consumption but also on past consumption of the individual's neighbors and individual past labor income.

Model

State Variables, Individual Labor Income, and Financial Market:

- We consider an economy with two state variables: non-tradable risky labor income $Y(t)$ and the risky stock price $S(t)$.
- We assume generic dynamics of individual labor income, driven by a Brownian motion $Z_Y(t)$.
- We assume the following dynamics for the stock price $S(t)$ and the price of a risk-less asset $B(t)$:

$$dS(t) = (r + \lambda_S \sigma_S) S(t)dt + \sigma_S S(t)dZ_S(t);$$
$$dB(t) = rB(t)dt;$$

  where $\lambda_S \in \mathbb{R}$ denotes the market price of stock return risk, $\sigma_S > 0$ models the stock return volatility, $Z_S(t)$ is a Brownian motion, and $r \in \mathbb{R}$ denotes the risk-less interest rate.
- We allow $Z_S(t)$ and $Z_Y(t)$ to be correlated, and denote their correlation coefficient by $\rho_{SY} \in [-1; +1]$.

Dynamic Budget Constraint

- Denote by $\omega(t)$ the share of pension wealth $F(t)$ invested in the risky stock at adult age t.
- The individual's dynamic budget constraint is given by

$$dF(t) = (r + \omega(t)(\mu_S - r_S))F(t)dt + \omega(t)\sigma_S F(t)dZ_S(t) + (Y(t) - c(t))dt.$$

- Pension wealth grows because of two reasons:
  (i) investment results;
  (ii) new savings $Y(t) - c(t)$.

Dynamic Optimization Problem

- The individual faces the following dynamic maximization problem:

$$\max_{c(t);\,!\,(t)} \quad E_0 \int_0^{T_D} e^{\ t} u(c(t) \quad h(t))\, dt$$

$$dh(t) = (\quad c(t) \quad (t)h(t))\, dt$$

$$dF(t) = (\, r + !\,(t) \ _S \ _S)\, F(t)dt + !\,(t) \ _S F(t)dZ_S(t)$$

$$+ (\, Y(t) \quad c(t))dt :$$

- Maximize expected lifetime utility subject to the reference level dynamics and the dynamic budget constraint.

4. Main Findings

- For the illustrations that follow, we rely where possible on parameter values from the existing literature.
- Our main implications remain qualitatively unchanged if we vary the values of the parameters within reasonable limits.

Main Finding III: Conservatism

- Non-tradable labor income risk and reference-dependent preferences lead to a conservative optimal portfolio strategy.

Welfare Costs

A strategy in which the savings rate does not respond excessively sensitive to a labor income shock can be quite costly in welfare terms.

true parameters	and	minimum welfare loss (in %)
0.05	0.1	38.04
0.1	0.2	35.08
0.2	0.3	30.13
0.3	0.4	26.04
0.4	0.5	23.52

Welfare costs are measured in terms of the relative decline in certainty equivalent consumption.

Data

- We obtain data from the U.S. Bureau of Labor Statistics (Consumer Expenditure Survey).
- We use monthly data on labor income and total expenditures.
- Our dataset runs from January 2020 to August 2021 (20 periods).
- Our dataset includes 15,381 unique individuals.

Heterogenous Response of Expenditures to Income Shocks

- Regression model:

$$\log c(t) = \log Y(t \quad 1) + (t):$$

- We divide the data into 3 income groups:
  - Low monthly gross incomes
  - Middle monthly gross incomes
  - High monthly gross incomes
- Coe cient estimates (all statistically signi cant):
  - $\hat{}_{\text{Low}} = 0:0270$
  - $\hat{}_{\text{Middle}} = 0:1200$
  - $\hat{}_{\text{High}} = 0:2159$
- Heterogeneous excess sensitivity.
- CRRA preferences do not predict any of this.

5. Conclusion

- We have explored the joint impact of reference-dependent preferences and non-tradable labor income risk.
- Three key findings:
  1. Excess sensitivity of optimal consumption and portfolio share to labor income shocks. Withdrawing pension wealth in a wide range of economic scenarios.
  2. Response is heterogeneous and heavily varies with ratio of consumption to reference level.
  3. Conservative consumption and investment strategies.
- Welfare losses can be as large as 35%.
- Findings remain intact in the case in which labor income shocks are not permanent.
- Findings are consistent with patterns in monthly savings data.
- To analyze the optimal policies and to determine the shadow price of labor income risk, we have developed a non-trivial solution procedure.

Thank you for your attention!