Roadmap

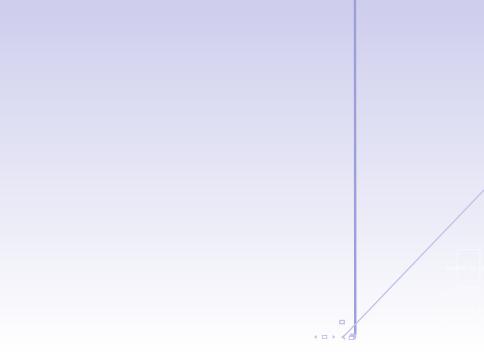
- Introduction
- 2 Literature review on Cooperative Games
- Cooperative Game with Multi-cooperation: Contributions
 - Theoretical results : Contributions
 - Application of obtained results to insurance market with an intermediate: Two main contributions
- Concluding remarks

Objective

De nition : Game, payo and Core

De nition

- A cooperative game (shortly CG): pN; q
 N: set of players (actors, economic agents)
 : mapping from P(N) to R satisfying pHq 0:
- ② A payo (allocation) of pN_{δ} $\mathbf{q}: x$ $px_1; x_2;$; $x_n \mathbf{q} \mathbf{P} \mathbb{R}^N$ satisfying e ciency, that is, $x \mathbf{p} N \mathbf{q}$ x_i $pN \mathbf{q}$.
- **3** A payo $x px_1; x_2; ; x_n q$ is :



Non emptiness of the core of the CG pN; q

De nition : balancedness coe cients

- A nonempty family of coalitions of N is balanced if there exists a sequence p TqT∈ of positive reals numbers satisfying :
 @i P N; T∈ : i∈T
- **2** pN; \mathbf{q} is balanced if for all balanced family of coalitions of N with balancedness coeficients \mathbf{p} $\tau \mathbf{q}_{T \in T}$, we have : $\tau_{T \in T} \mathbf{p}_{T} \mathbf{q} \mathbf{x} \mathbf{p}_{T} \mathbf{q}_{T}$
- $\mathbf{p}_{T}\mathbf{q}_{T\in}$: balancedness coe cients of :

NSC for non-emptiness of the core or stability of CG pN;

Theorem (Bondareva 1963 and Shapley 1967)

A CG pN; q is stable if and only if it is balanced.

Cooperative Game with Multi-cooperation

De nition of SCG and SCGMC

- A structured cooperative game (shortly SCG): pN; ;Sq where pN; q is a CG and S is a recovering of N:
- Cooperative game with multiple cooperations (shortly SCGMC): The SCG pN; ;Sq where some elements of S are not pairwise disjoints, that is, S is not a partition of N:

Payo of a Cooperative Game with Multi-cooperation

Let pN; ; Sq be a SCG with $S = \mathbf{t}R_1$; ...; $R_m\mathbf{u}$.

Payo of pN; ; Sq: a sequence x $tx_{i;k}u_{i\in N; 1 \le k \le m}$ of real numbers satisfying e ciency on coalitions of the recovering S; that is,

@k P t1;2; ;
$$mu$$
; xpR_kq $x_{i;k}$ pR_kq : (1)

Characterization of elements of CpN; ; Sq

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Let x = \mathbf{t} x_{i,k} \mathbf{u}_{i \in N; 1 \le k \le m} be a payo of \mathbf{p} N; ; S \mathbf{q} be a SCG with
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S $\mathbf{t}R_1; ...; R_m \mathbf{u}$:

x P CpN; ; Sq if and only if

@
$$k$$
 P t1/2; /mu/ @ S ,, R_k : $X_{i;k}$ ¥ p S q:

De nition

Insurance market model: Coalitions gains (inspired from Eckardt, 2007)

Gross gain of in exchange between the insurer and the insured a_i and the characteristic function

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• Direct cooperation : V_{a_i}^D C_{a_i}^D • \mathsf{p} T_{insured}^D T_{insured}^D : • Intermediary exchange : V_{a_i}^I C_{a_i}^I • \mathsf{p} T_{insured}^I T_{insured}^I • T_{insured}^I
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pKq is given by :
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if A P K; Da_i P K and I R K;
                           if A P K : Da_i P K and I P K :
                           otherwise:
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(3)



Insurance market model: second contribution with Game Theory approach

ISGC with intermediary exchange

The ISGC pN; ; Sq is in an intermediary exchange if $I P \underset{K \in S}{\mathbf{X}} K$:

Proposition: Fourth main result

Let (N; :S) be a ISCG with an intermediary exchange.

The following assertions are equivalent:

- Condition IE_2 is satis ed and the total transaction cost mapping T is sub-additive.
- ② CpN; ;Sq H:

$$T: 2^{N_a} \widetilde{\mathbf{N}} \mathbb{R}^+$$
 where @F P 2^{N_a} ; T pFq $T_F^I = T_{ins;F}^I$.

T is sub-additive, that is,

 $@F:GP2^{N_a}:FXG$ H ùñ TpFYGq = TpFq TpGq:

Summary

Introduce the rst concepts of a cooperative game with multiple cooperations: recovery of all players, payo, dominance between payo s, core of the game and game stabilitdom.955 Td [(co)-27(op)-26(eTd)

Summary

Introduce the rst concepts of a cooperative game with multiple cooperations: recovery of all players, payo, dominance between payo s, core of the game and game stability.

Characterize the elements of the classical core of the new game, Establish stability of cooperative game with multiple cooperations.

Establish the conditions under which it would be in the insurer's interest to draw up each contract with a policyholder (individual contract) through an intermediary.

Open question

Study an insurance market with several insurance companies and several intermediaries.

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Thanks

- The World Academy of Sciences (TWAS)
- UNSW Business School
- ASTIN