



teacher-fu

teacher-fu



Roadmap

- 1 Introduction
- 2 Literature review on Cooperative Games
- 3 Cooperative Game with Multi-cooperation : Contributions
 - Theoretical results : Contributions
 - Application of obtained results to insurance market with an intermediate : Two main contributions
- 4 Concluding remarks

teamwork

cooperative

Objective

Definition : Game, payoff and Core

Definition

- 1 A cooperative game (shortly CG) : $(N; v)$
 N : set of players (actors, economic agents)
 v : mapping from $P(N)$ to \mathbb{R} satisfying $v(\emptyset) = 0$;
- 2 A payoff (allocation) of $(N; v)$: $x = (x_1; x_2; \dots; x_n) \in \mathbb{R}^N$ satisfying efficiency, that is, $\sum_{i \in N} x_i = v(N)$.
- 3 A payoff $x = (x_1; x_2; \dots; x_n)$ is :

Non emptiness of the core of the CG $pN; q$

Definition : balancedness coefficients

- 1 A nonempty family of coalitions \mathcal{T} of N is balanced if there exists a sequence $(p_T)_{T \in \mathcal{T}}$ of positive real numbers satisfying :

$$\sum_{T \in \mathcal{T}; i \in T} p_T = 1;$$

$$\forall i \in N;$$
- 2 $pN; q$ is balanced if for all balanced family of coalitions \mathcal{T} of N with balancedness coefficients $(p_T)_{T \in \mathcal{T}}$, we have : $\sum_{T \in \mathcal{T}} p_T q \preceq pNq$.

$(p_T)_{T \in \mathcal{T}}$: balancedness coefficients of \mathcal{T} :

NSC for non-emptiness of the core or stability of CG $pN; q$

Theorem (Bondareva 1963 and Shapley 1967)

A CG $pN; q$ is stable if and only if it is balanced.

teamwork

teamwork

Cooperative Game with Multi-cooperation

Definition of SCG and SCGMC

- 1 A structured cooperative game (shortly SCG) : $(N; \mathcal{S}; v)$ where $(N; v)$ is a CG and \mathcal{S} is a recovering of N ;
- 2 Cooperative game with multiple cooperations (shortly SCGMC) : The SCG $(N; \mathcal{S}; v)$ where some elements of \mathcal{S} are not pairwise disjoint, that is, \mathcal{S} is not a partition of N ;

Payoff of a Cooperative Game with Multi-cooperation

Let $(N; \mathcal{S}; v)$ be a SCG with $\mathcal{S} = \{R_1, \dots, R_m\}$.

Payoff of $(N; \mathcal{S}; v)$: a sequence $x = (x_{i;k})_{i \in N; 1 \leq k \leq m}$ of real numbers satisfying efficiency on coalitions of the recovering \mathcal{S} ; that is,

$$\sum_{i \in R_k} x_{i;k} = v(R_k) \quad \forall k \in \{1, 2, \dots, m\} \quad (1)$$

Cooperative Game with Multi-cooperation : 1st main result

Characterization of elements of $CpN; ;Sq$

Let $x = (x_{i;k})_{i \in N; 1 \leq k \leq m}$ be a payoff of $pN; ;Sq$ be a SCG with $S = (R_1; \dots; R_m)u$
 $x \in CpN; ;Sq$ if and only if

$$\forall k \in \{1; 2; \dots; m\}; \forall S' \subseteq S, R_k \cap S' \neq \emptyset \implies x_{i;k} \geq p(S')$$

Cooperative Game with Multi-cooperation : On the path of stability

De nition

Insurance market model : Coalitions gains (inspired from Eckardt, 2007)

Gross gain of in exchange between the insurer and the insured a_i and the characteristic function

- Direct cooperation : $V_{a_i}^D \quad C_{a_i}^D \quad p T_{insured}^D \quad T_{insurer}^D q$
- Intermediary exchange : $V_{a_i}^I \quad C_{a_i}^I \quad p T_{insured}^I \quad T_{insurer}^I q$

pKq is given by :

$$\begin{aligned}
 & \begin{cases} \$ \\ \& \\ \% \\ 0 \end{cases} \begin{cases} \circ \\ \\ \\ \end{cases} \\
 & \begin{cases} p V_a^D \quad C_a^D q \quad p T_{K \cap N_a}^D \quad T_{ins; K \cap N_a}^D q & \text{if } A \in K; Da_i \in K \text{ and } I \in K; \\ p V_a^I \quad C_a^I q \quad p T_{K \cap N_a}^I \quad T_{ins; K \cap N_a}^I q & \text{if } A \in K; Da_i \in K \text{ and } I \in P K; \\ 0 & \text{otherwise:} \end{cases} \\
 & \hspace{15em} (3)
 \end{aligned}$$

Teamwork

Cooperation

Insurance market model : second contribution with Game Theory approach

ISGC with intermediary exchange

The ISGC $(N; \cdot; S; q)$ is in an intermediary exchange if $\exists P \times_{K \in S} K :$

Proposition : Fourth main result

Let $(N; \cdot; S)$ be a ISGC with an intermediary exchange.

The following assertions are equivalent :

- 1 Condition IE_2 is satisfied and the total transaction cost mapping T is sub-additive.
- 2 $C_p(N; \cdot; S; q) = H :$

$T : 2^{N_a} \rightarrow \mathbb{R}^+$ where $\forall F, P \in 2^{N_a}; T_p F, q = T_F^I = T_{ins; F}^I.$

T is sub-additive, that is,

$\forall F, G \in 2^{N_a}; F \cap G = \emptyset \Rightarrow T_p(F \cup G), q \leq T_p F, q + T_p G, q :$

Summary

Introduce the first concepts of a cooperative game with multiple coalitions : recovery of all players, payoff, dominance between payoffs, core of the game and game stability.

Summary

Introduce the first concepts of a cooperative game with multiple cooperations : recovery of all players, payoff, dominance between payoffs, core of the game and game stability.

Characterize the elements of the classical core of the new game, Establish stability of cooperative game with multiple cooperations.

Establish the conditions under which it would be in the insurer's interest to draw up each contract with a policyholder (individual contract) through an intermediary.

Open question

Study an insurance market with several insurance companies and several intermediaries.

References

- [1] Asimit A.V. and Boonen T.J. (2018) *Insurance with multiple insurers : A game-theoretic approach*. European Journal of Operational Research 267(2), 778-790.
- [2] Aumann R.J. and Dreze J.H. (1974) *Cooperative Games with Coalition Structures*. International journal of Game Theory 4, 217-237.
- [3] Bondareva O. (1962) The theory of the core in an n-person game. *Vestnik Leningrad. Univ* 13, 141-142.
- [4] Bondareva O.N. (1963) Some applications of linear programming methods to the theory of cooperative games. *Problemy kibernetiki* 10, 119-139.
- [5] Gillies D.B. (1953) Some theorems on n-person games. Ph.D. thesis, Princeton University, Princeton, N.J.
- [6] Eckardt M. (2007) Insurance intermediation : An Economic Analysis of the Information Services Market, *In Contributions to Economics Series 22*, Springer, Physica-Verlag.



teamer-tu
eamer-tu

Thanks

- The World Academy of Sciences (TWAS)
- UNSW Business School
- ASTIN