Gaussian Process-Based Mortality Monitoring using Multivariate Cumulative Sum Procedures

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Monitoring insurance processes

Monitoring mortality rates is crucial for the risk management of life insurance.

Challenges:

- **Quickest detection:** In a rapidly changing environment, actuarial assumptions should be monitored quickly and efficiently.
  → Real-time sequential detection
- **Correlation:** Mortality data often exhibit interdependencies between different age groups or cohorts.
  → Gaussian Process (GP) regression
- **Multivariate detection:** Univariate detection methods ignore the complex dependence structure, limiting their effectiveness.
  → MCUSUM algorithm

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This presentation

- Proposed approach:
  - Forecasting: Mortality forecasting based on GP regression.
  - **MCUSUM monitoring:** Tracks differences between predicted and observed mortality rates, enabling real-time change detection.
- Which change?
  - Change of level by tracking mortality rates.
  - Change of trend by tracking mortality improvements.
- Empirical analysis for France, Japan, Canada, and the USA.
- The MCUSUM shows quicker detection to univariate alternatives that ignore dependence.

Gaussian process for mortality forecasting

- Training set: (x^i, y^i) $(i = 1, \ldots, n)$.
  - In our case: $x^i = (x^i_{\text{age}}, x^i_{\text{year}})$ and $y^i = \log(D^i/E^i)$.
  - Age: M age-groups, e.g. $z_1 = [50; 55); z_2 = [55; 60); \ldots; z_M = [85; 90)$.
  - T years: [1980,2020].
- Gaussian process:

$$f(x) \quad N\left(m(x), k(x, x)\right),$$

where $k(x, x)$ is the covariance matrix.

- Completely characterized by mean function $m(x)$ and covariance/kernel function $k(x, x\ )$.
- Key reference: Ludkovski et al. (2018).

Gaussian process for mortality forecasting

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Gaussian process for mortality forecasting

- GP posterior distribution is multivariate normal.
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The MCUSUM for multivariate normal (continued)

- For mortality monitoring, log death rates follow
  - In-control process: $N(\boldsymbol{\mu_1}, \Sigma)$.
  - *Out-of-control process*: $N(\boldsymbol{\mu_2}, \Sigma)$.
- The MCUSUM is

$$S_t = \max\left(S_{t-1} + (\boldsymbol{\mu_2} - \boldsymbol{\mu_1})\ \Sigma^{-1}\left(y^t - \boldsymbol{\mu_1}\right) - \frac{1}{2}(\boldsymbol{\mu_2} - \boldsymbol{\mu_1})\ \Sigma^{-1}(\boldsymbol{\mu_2} - \boldsymbol{\mu_1}), 0\right).$$

What type of change?

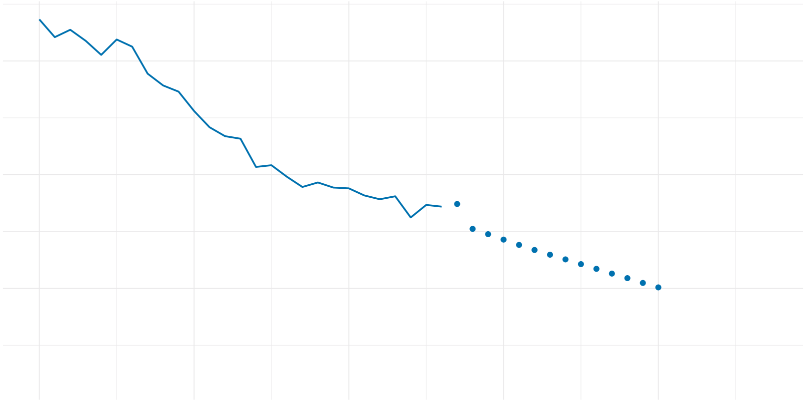

Figure: Mortality rate at age 65 in France with Lee-Carter forecasts, change of level and change of trend with a change point in 2017.

Change of level Detection

- The change-point model for level change detection can be expressed as

$$\mathbb{E}[\log(\mu_t)] \quad \begin{cases} m_t & \text{for } i = 1, \ldots, \tau \\ \overline{m_t} & \text{for } i = \tau + 1, \ldots \end{cases}$$

where e.g. $\overline{m_t} = m_t + \log(\alpha)1$ with $\alpha = 0.9$ (longevity risk).

- The generalized MCUSUM is defined by:

$$S_t = \max\left(S_{t-1} + (\overline{m_t} - m_t)' \Sigma_t^{-1} (y^t - m_t) - \frac{1}{2}(\overline{m_t} - m_t)' \Sigma_t^{-1} (\overline{m_t} - m_t), 0\right),$$

where

1. y^t is the vector of observed log death rates.
2. m_t and Σ_t are the mean and covariance from GP-based forecasts.

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Change of trend detection

- The change-point model for trend change detection can then be expressed as

$$\mathbb{E}\left[\Delta \log(\mu_t)\right] \quad \begin{cases} m_t^l & \text{for } i = 1, \ldots, \tau \\ \overline{m}_t^l & \text{for } i = \tau + 1, \ldots \end{cases}$$

where

1. **Mortality improvements:** $\Delta \log(\mu_t) = \log(\mu_t) - \log(\mu_{t-1})$
2. **Trend change:** $\overline{m}_t^l = \log(\exp(m_t^l) - \alpha)$

- How to fix the threshold L?

$$\mathbb{P}\left[\max_{1 \leq i \leq T} S_i \geq L \mid \text{no change}\right] = \alpha,$$

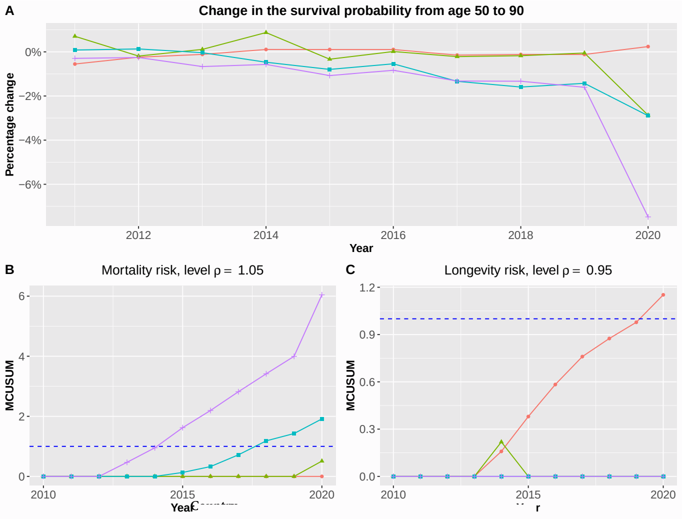
determined by simulations.

threshold?

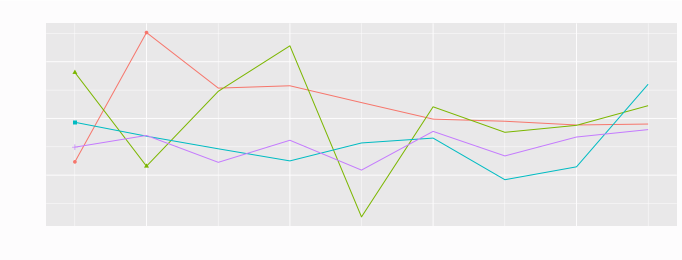
Empirical analysis

- Countries: France, Canada, USA and Japan.
- Ages: 50-89 by 5-year age tranches.
- Years:
  1. Estimation: 1991-2010.
  2. Detection: 2011-2020.
- Detection types:
  1.

Empirical analysis: change of level



Empirical analysis: change of trend



MCUSUM vs univariate CUSUM charts

What is the **added value** of the MCUSUM?

- Standard age-period-cohort models assume perfect correlation

MCUSUM vs univariate CUSUM charts

What is the **added value** of the MCUSUM?

- Standard age-period-cohort models assume perfect correlation, e.g. for the Lee-Carter model:

$$\log(\mu$$

MCUSUM vs univariate CUSUM charts

The comonomotonic CUSUM is defined as

$$S_t^c = \max\left(S_{t-1}^c + (\overline{\mu}_t - \mu_t)\frac{(s_t - \mu_t)}{\sigma_t} - \frac{1}{2}\frac{(\overline{\mu}_t - \mu_t)^2}{\sigma_t}, 0\right),$$

with

$$\mu_t = \sum_{x=1}^{M} m_{i,t} \qquad \sigma_t = \sum_{x=1}^{M} \sigma_{i,t}$$

$$\overline{\mu}_t = \mu_t + M\log(\alpha) \qquad s_t = \sum_{x=1}^{M} y_{i,t}$$

with $m_{i,t}$ and $\sigma_{i,t}$, the mean and standard deviations of the i-th component of the log death rates vector $y_t = (y_{1,t}, \ldots, y_{M,t})$.

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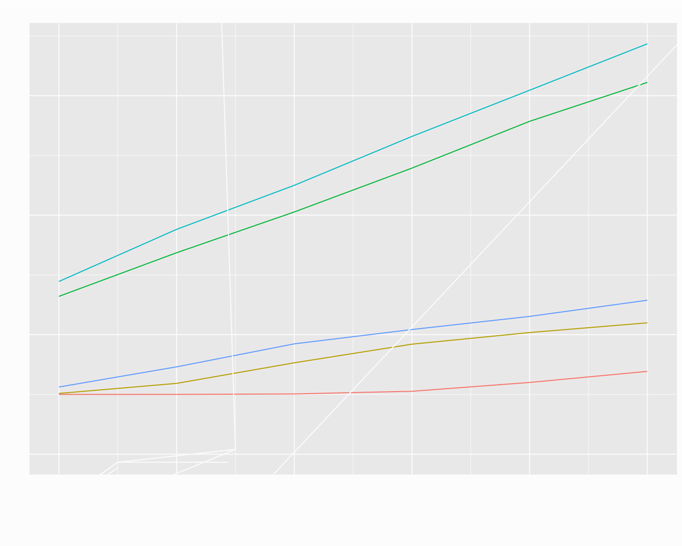
Comparison of the MCUSUM and C-CUSUM charts



Conclusion

- GP-based mortality forecasts combined with the MCUSUM detection rule provide several benefits:
  1. Capture the dependence between age classes.
  2. **Efficient real-time multivariate monitoring** for e.g.
     - ★ Change of level.
     - ★ Change of trend.
  3. Detection of longevity risk in Japan and mortality risk in USA and Canada over the 10-year period 2011-2020.
  4. **Outperformance compared to univariate control charts** that ignore the dependence structure.

Thank you for your attention! Any questions?

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Conclusion

- GP-based mortality forecasts combined with the MCUSUM detection rule provide several benefits:
    1. 18/18

Extra slides *just in case*

Figure: Percentage change between observed and GP-predicted death rates by age tranches for Japanese males.

Extra slides *just in case*

Extra slides *just in case*

	[50,55)	[55,60)	[60,65)	[65,70)	...	[85,90)
[50; 55)	1	ρ	$\rho/2$	0	0	0
[55; 60)	ρ	1	ρ	$\rho/2$	0	0
[60,65)	$\rho/2$	ρ	1	ρ	$\rho/2$	0
[65,70)	0	$\rho/2$	ρ	1	ρ	$\rho/2$
...	0	0	$\rho/2$	ρ	1	ρ
[85; 90).	0	0	0	$\rho/2$	ρ	1

Table: Correlation matrix between age tranches used for the simulation study.

Extra slides *just in case*

Figure: Estimated correlation matrix for Japanese male death rates in 2011.

Ludkovski, M., Risk, J. & Zail, H. (2018), 'Gaussian process models for mortality rates and improvement factors', *ASTIN Bulletin: The Journal of the IAA*