

SOCW7851

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7ci fgY 8YW] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

I g]b['Uˈf][\hg 'VUgYX 'UddfcUW\žh\Y 'Wti fgY 'YI Ua]bYg'h\Y 'bUhi fY 'cZWta a i b]hmXYj Y`cda Ybhi]b 'Ub]bhYfbUhjcbU``UbX 'bUhjcbU``WtbhYI hzXfUk]b['cb'UʻfUb[Y`cZh\YcfYh]WU``UbX`WUgY 'ghi Xma UhYf]U`g''`H\Y Wti fgY 'Wtj Yfg'U`Vf]YZ\]ghcfmcZWta a i b]hmXYj Y`cda YbhYh\Y 'W\Ub[]b['bUhi fY 'cZWta a i b]hmk cf_žh\Y WtbWYdhg'cZW 'hi fY']b 'fY`Uhjcb'hc 'Wta a i b]hmk cf_jb 'XYj Y`cd]b['gcWjYhjYgžUbX'h\Y 'XmbUa]Wg'cZ Wta a i b]WUhjcbž'|bWi X]b['Wfcgg'W 'hi fU` Wta a i b]WUhjcbž'k cf_]b['k]h\ ']bhYfdfYhYfgž'\i a Ub'f][\hg XcW a YbhUhjcb'UbX'UXj cWUwh'8]ZZYfYbh]XYc`c[]WU``UddfcUW\Yg'hc 'Wta a i b]hmk cf_'UfY YI Ua]bYX'UbX'Ub UbU`mg]g'cZh\Y']a dUWhg'UbX'ci hWta Yg'h\Uhih\YgY'UddfcUW\Yg'a][\hi\Uj Y'cb'Wta a i b]hjYg']g'YI d`cfYX'' 5`hYfbUhjj Y'a cXY`g'cZfYgYUfW\Z'Wtbgi `hUhjcb'UbX'bYYXg'UbU`mg]gž'd`Ubb]b['UbX'gi ghU]bUV`Y'gYfj]WW XY`]j YfmUfY'YI Ua]bYX']b'cfXYf'hc 'UXXfYgg'W\Ub[]b['UbX'Ya Yf[]b['bYYXg''

7ci fgY @YUfb]b['Ci hWca Yg

- %" ⇒XYbhjZmh\Y``]b_g`VYrk YYb`Wca a i b]lmiXYj Y`cda YbhUbX`U`f][\hg`VUgYX`ZfUa Yk cf_`Zcf`GcWJU` XYj Y`cda Ybh
- &" @cWUHY 7 ca a i b]lmi8 Yj Y`cda Ybh]b`k]h\]b`U`VfcUXYf`ZfUa Yk cf_cZWca a i b]lmik cf_
- '" =XYbhZmh\Y]a dUWhcZ; YbXYfUbX7i`h fYcb7ca a i b]hm8YjY`cda YbhdfcYWhq
- ("⇒XYbhZmh\Y'fc`Y'cZUXjcWWmjb'Wcaaib]lmxYjY`cd

HYUWX]b['GhfUhY[]Yq'

H\]g`]g`U`dcgh[[fUXi UhY`Wci fgYz`UbX`ghi XYbhg`UfY`Yl dYWYX`hc`i bXYffU_Y`U`g][b]Z]WJbh`Ua ci bh`cZ
]bXYdYbXYbhfYgYUfW\]bhc`h\Y`]ggi Yg`i bXYf`Wcbg]XYfUhjcb"`H\YfY`]g`Ub`Yl dYWfUhjcb`h\Uhighi XYbhg`k]```i gY
Ub`UbU`mhj\WJ``UddfcUW\`hc`h\Y]f`k cf_z`UbX`g\ck 'h\Y`UV]`]hm`hc`Udd`mh\Ycfm'UbX`hc`h\]b_`Wf]hj\WJ``m'`9j]XYbW\
cZh\]g`a i ghVY`dfcj]XYX`]b`U``k f]hhYb`k cf_`gi Va]hhYX`Zcf`UggYgga Ybh'

5 b 'UXi 'h'UddfcUWk 'hc 'YUfb]b[']g'hU_Yb/h\ Uh]g'gh XYbhg'UfY 'fYgdcbg]V'Y 'Zcf'h\ Y'dUWk 'UbX'Ua ci bhcZ fYUX]b['UbX'YI hfU 'fYgYUfWk 'h\ YmXc'Ug'k Y``'Ug'UhYbXUbWk 'UhWUggYgž'UbX'Zcf'fU]g]b['k]h\ 'h\ Y'WccfX]bUhcf]ggi Yg'cf'dfcV'Ya g'k]h\ 'h\ Y]f'`YUfb]b["'Gh XYbhg'UfY 'U'gc'YI dYWkYX'hc'hU_Y'U'dUfh]W]dUhcfmUddfcUWk']b WUggYg'gc'h\ Uh_bck 'YX[Y']g'g\ UfYXž'ei Ygh]cbYXž'XYVUhYX'UbX'Uddfcdf]UhYX']bX]j]Xi U``mUbX']b'[fci dg" GYa]bUfg'UfY'dfY! fYwcfXYX'UbX'Uj U]`UV'Y'Zcf'ghfYUa]b[#Xck b`cUX'Zfca 'A ccX'Yž'UbX'h\ YfY']g'U j c'i bhUfmcbY!\ci f'hi hcf]U``YUWk 'k YY_'hc'\Y'd'gh XYbhg'k]h\ 'Ubmei Ygh]cbgž'Wca a Ybhg'UbX'cddcfhi b]hmZcf X]gW gg]cb'hc'h\ Y' k YY_fg'hcd]W'DUfh]W]dUh]cb']g'bch[fUXYX"'< ck Yj Yfž']h']g'U'Wci fgY'YI dYWfUh]cb'h\ Uh fYei]fYX'fYUX]b[g'k]```VY'fYUX'VmU``'dUfh]W]dUbhg'YUWk 'k YY_"

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A]X!HYfa 'F YZYW¶j Y'9ggUm	&) ।	&, #\$*#&\$&%`%%\$\$`DA	

7\ccgY`Zfca`th\Y``]ghcZei Ygh]cbg`cf`WfYUhY`mci f`ck b`fd`YUgY`Wcbgi`thk]h\`th\Y`Wci fgY`Wcbj YbYf`]Z
W\ccg]b[`mci f`ck b`ei Ygh]cbt"`H\Y`dUdYf`]g`i d`hc`' \$\$\$`k cfXg`]b`UXX]h]cb`hc`fYZYfYbWYg`CF`mci `\Uj Y`th\Y
cdh]cb`cZ\Uj]b[`U`th_Y`\ca Y`hYgh]bghYUX"`H\Y`th_Y`\ca Y`hYghk]``[c`Zcf`Z]j Y`XUmgž`UbX`mci `bYYX`hc`k f]hY
th\fYY`g\cfh`YggUmg`f%\$\$\$`k cfXg`dYf`g\cfh`YggUmt"`H\Y`ei Ygh]cbg`k]```VY`fY`YUgYX`cbY`XUmVYZcfY`thYgh
ghUffg"`K f]hhYb`ZYYXVUW_UbX`U`a Uf_`k]``VY`dfcj]XYX`k]h\]b`&`k YY_g`cZgi Va]gg]cb"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYH]b[."H\]g"Ugg][ba Ybh]g"gi Va]hYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \\ \end{tabular}$

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DfYgWf]VYX F Ygci fWYg

FYei]fYXfYUX]b[g'k]``VY'UjU]`UV`Y'cb'AccX`Y"`

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 $\label{eq:final_problem} F\ Y\ W\ ca\ a\ Y\ b\ X\ Y\ T\ Y\ T\ U\ g\ k\]\ ```VY\ U\ U\ U\ U\ Y\ C\ b\ A\ C\ C\ X\ Y\ ''$

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7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Ghi XYbhZYXVUW_cZYj U`i Uhjcb`]g`Wfi WjU`hc`XYj Y`cd`UbX`]a dfcj Y`h\]g`Wti fgY`Z fh\Yf`]b`gi VgYei YbhmYUfg" A m9I dYf]YbWf`gi fj Ymk]``VY`WtbXi WhYXhck UfXg`h\Y`YbX`cZh\Y`hYfa "`5hK YY_'' žh\YfYk]``VY`U``Wti fgY\YU`h\`W\Y\W_``k]h\`h\Y`WUgg"`H\]g`k]```VY`Ub`cddcfhi b]hmZcf`ghi XYbhg`hc`dfcj]XY`ZYYXVU_UbX gi [[Ygh]cbg`cb``YWhi fYžWtbhYbhz`fYUX]b[gžhi hcf]U``UWhjj]h]Yg`UbX`ch\Yf`h\]b[g`k\]W\`k]``Yb\UbWf``YUfb]b[YI dYf]YbWf"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

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- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`ghi XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc']W]Yg/
- YI Ua]bUh]cb dfcWYXi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/