

ARTS2815

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GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
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7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

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7ci fgY @YUfb]b[Ci hWca Yg

- %" 5 bU`mgY`X]ZYfYbh'UddfcUW\Yg`hc`h\Y`Wcbglfi Wf]cb`cZfY[]cbg`UbX`fY[]cbU`]ga `UbX`Udd`mih\YgY`hc h\Y`WcbWldhcZh\Y`\=\Xc!DUV]ZJW
- &" Ci h]bY'q][b]Z\WbhWcbhYa dcfUfm]qqi Yq'UbX'W\U``Yb[Yq']b't\Y'+bXc!DUW\Z\W
- ' " 9j U`i UHY`H\Y`gVXc`Uf`m`]HYfUH fY`cb`_Ym]gqi Yg`UbX`VXU``Yb[Yg`]b`H\Y`±bXc!DUV]ZW
- ("7cbXi Wh]bXYdYbXYbhUbX gWkc`UfmfYgYUfWk"
-) " 7 cbgffi WhiUb Yj]XYbWY!VUgYX Uf[i a YbhVch\ cfU`mUbX]b k f]h]b["

HYUWX]b[GhfUhY[]Yg

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
7`UggʻZUWj`]hUhjcbʻ	&) ।	Bch5dd`]WUV`Y	&ž˙' ž˙)
7 f]f]WU`F Yj]Yk '	&) ı	\$&#\$+#&\$&\$`\$).\$\$`DA</td><td>'Ž'(Ž')</td></tr><tr><td>A Uʻcfʻ9ggUmi</td><td>) \$1</td><td>% #\$, #&\$&\$`\$).\$\$`DA</td><td>%ž'&ž'(ž')</td></tr></tbody></table>	

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% 7`Ugg ZUWj`]hUhjcb

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

%!%) 'a]b 'WUgg ZJW]']HUh]cb 'VUgYX 'cb 'Ugg][bYX 'fYUX]b[g ']b '[fci dg 'cZ' !('gh XYbhg Zc ``ck YX 'VmX]gW gg]cb" 5 ```gh XYbhg k]```fYWY]j Y h\Y 'gUa Y 'a Uf_"

Gh XYbhg k]``fYWY]j Y k f]hhYb ZYYXVUW_k]h\]b '% k cf_]b['XUmg cZh\Y]f ZUW]r]hUh]cb"

5XX]h]cbU`XYhU]`g.

Mci 'UfY 'fYei]fYX'hc 'gYfj Y'Ug'"WX]YZ'X]gW ggUbhg" cZh\Y'fYUX]b[g']b'h\Y'gYa]bUf'Uh'YUghcbW'Xi f]b['h\Y Wci fgY"'8 YdYbX]b['cb'h\Y'g]nY'cZh\Y'WUggžYUWX'k YY_'h\fYY'cf'Zci f'ghi XYbhg'k]```k cf_'Ug'U'hYUa 'UbX 'YUX'cZh\Y'X]gW gg]cb"'H\Y'dfYgYbhYfg'a i gha U_Y'U'%)!&\$'a]b'dfYgYbhUhcb'cZh\Y'k YY_'g'fYUX]b[g'UbX ZUW]']hUhY'X]gW gg]cb"'F Uh\Yf'h\Ub'g]a d`mgi a a Uf]n]b['th\Y'fYUX]b[g'i bXYf'ei Ygh]cb'fdfYgi a UV'mU` k]``\Uj Y'fYUX']h\Z'h\Y'X]gW gg]cb'ZUW]']hUhcfg'g\ci `X'Wf]h]WU``mYb[U[Y'k]h\Th\Y'a UhYf]U`'UbX'hYUgY ci hgdYW]ZJWei Ygh]cbg'cf'dfcV'Ya g'h\UhUddYUf'k cfh\mcZ'UhYbh]cb'UbX'WUb'VY'U'VUg]g'Zcf X]gW gg]cb"'5``ghi XYbhg']b'U'[fci d'XY'Y[UhYX'k]h\Th\Y'fYgdcbg]V]`]hmhc'`YUX'h\Y'X]gW gg]cb'k]``fYWY]j Y h\Y'gUa Y'a Uf_"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh'& '7f]h]WU`FYj]Yk

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'+) \$'k cfXg

8YHJ]`g.

 $\label{thm:condition} $$HY''Wf]hW'''fYj]Yk''fYei]fYg'ghi XYbhg'hc'gY'YWhcbY'UWUXYa]WUfh]WY'Zfca''U'k YY_fg'fYUX]b['UbX XYa cbghfUhY'U'WcgY'Yb['U['Ya Ybh'UbX'UbU'mg]g'k]h\'h\Y'a U]b''Uf['i a Ybhg'UbX'WU]a g'cZh\Uh'Ufh]WY'''H\Y k cfX''Yb['h\'cZh\Y'fYj]Yk']g'+) $'k cfXg'''}$

: YYXVUW_. `Ghi XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW_`UbX`U`bi a Yf]WU``[fUXY`k]h\]b`hk c`k YY_g`cZgi Va]gg]cb h\fci [\`h\Y`l b]j Yfg]hmfg`@YUfb]b[`A UbU[Ya YbhGmghYa `fl@A GŁ"`

5XX]h]cbU`XYhJ]`g.

Mci 'Ufy 'fyei]fyx'hc 'dfydUfy'U'"fyUWfjcb'd]YWYZ"k\]WX 'g\ci `X'dfygYbhmci f`fygdcbgY'hcz'UbX'Wf]rjei Y'cZz h\Y'a U'cf'Uf[i a Ybhg'Zfca 'U'k YY_'g'fyUX]b["'Mci 'WUb 'gY'YWfih\Y'k YY_'cZmci f']bhYfyghhc fYZYWficbz'Vi h mci f`gY`YWfjcb'g\ci `X'VY'X]ZZYfYbhZfca 'h\Y'k YY_'cZmci f`WUgg'ZUWj']hUhjcb'f5gg][ba Ybh'%±"'H\]g'YggUm]g Xi Y'cb'>i `m&"'Mci 'k]``fYWY]j Y'k f]hhYb'ZYYXVUW_'k]h\]b'%\$'k cf_]b['XUmg"'H\Y'ZYYXVUW_'g\YYh#fi Vf]Wk]` VY'Uj U]`UVY'hc'ghi XYbhg'Uhh\Y'ghUfhcZh\Y'Wci fgY'gc'h\Uhh\YmWUb'k cf_hck UfXg'gdYWjZYX'ghUbXUfXg"

 $\label{eq:higher} \mbox{Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b q]a]`Uf]hmfYdcfhg"$

5ggYgga Ybh' . A U'cf 9ggUm

GHJfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &) \$\$ k cfXg

8YHJ]`g.

H\Y'YggUmi]g'h\Y'Z|bU`'UggYgga YbhcZh\Y'Wci fgY"

#ifYei]fYgʻghi XYbhgʻhcʻXYa cbghfuhYʻYb[U[Ya Ybhk]h\ʻh\YʻWcfYʻh\Ya YgʻUbXʻ]ggi YgʻfU]gYXʻ]bʻh\YʻWci fgY UbXʻh\Y]fʻUV]`]mhcʻWcbghfi WhʻUʻdYfgi Ug]j YʻUbXʻk Y``!Yj]XYbWXʻUf[i a Ybhʻh\Yʻk cfX``Yb[h\ʻcZh\YʻYggUm]g &) \$\$`k cfXg''

: YYXVUW_. `Ghi XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW_`UbX`U`bi a Yf]WU``[fUXY`k]h\]b`hk c`k YY_g`cZgi Va]gg]cb h\fci [\`h\Y`I b]j Yfg]hmfg`@YUfb]b[`A UbU[Ya YbhGmghYa `f@A GŁ"'H\Y`ZYYXVUW_`g\YYh#fi Vf]Wk]```VY Uj U]`UV`Y`hc`ghi XYbhg`Uhh\Y`ghUfhcZh\Y`Wci fgY`gc`h\Uhh\YmWUb`k cf_`hck UfXg`gdYW]ZJYX`ghUbXUfXg"

5XX]h]cbU`XYhU]`g.

 $\label{thm:condition} $$H\g^2_bU^*YggUm^Wb^*VY^cb^*U^*cd^WcZmcif^]bhYfYghVihVUgYX^cb^*cf^*fY^UhYX^hc^hXY^WcbhYbhcZhXY^WcifgY^UbX^*cf^*h^Y^WcbhYbhcZhXY^WcifgY^UhX^hc^hXY^WcifgY^UhX^hC^hXY^WcifgY^UhX^hC^hXY^HX^fa~z^*mcig^ci^X^hX]b_UVcihU^*hcd^WcZmcif^]bhYfYghUbX^\ck^]hWb^*VY^UbU^*mYX^"H\Y^ZgUmdfcdcgU^*bYYXg^hc^VYX]gW^gYX^k]h^h^hXY^]bghfi Whcf^Xif]b[^hX^*cZ]W^*\cifg^cf^*j]U^YaU]^"H\Y^Z]bU^*YggUm^]g^XiY^cb^5i[igh %""$

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY "hi fb]h "UbX"h Xybhg"Xc "bchgYY "bch$

5 hm/b XUb WY F Yei]fYa Ybhg

 $\label{eq:chain_formwaggygf} 6ch ``YWh fyg`UbX'hi hcf]U`g`UfY`a UbXUhcfmWUggYg`]b`5FHG\&, \%) `]b`H\&ž`&$\&\$"$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh
K YY_'%'' %'A Um!'('>i bY	@YWi fY	CjYfj]Yk:/:@c[]gh]Wg:cZ7cifgY.:Dc`]h]Wg:UbX GYWif]hmi]b:h\Y:+bXc!DUW]Z]W'X]ZZYfYbh`YjY`g:cZ UbU`mg]g/aU'cf:UWhcfgi]b:h\Y:+bXc!DUW]Z]WfY[]cb"''
	Hi hcf]U`	Dc`]hJWg`UbX`GYW`f]lm`]b`h\Y`&Xc!DUWJZJW'X]ZZYfYbh `Yj Y`g`cZUbU`mg]g/a U'cf`UWcfg`]b`h\Y`&Xc!DUWJZJW fY[]cb"`8]ghf]Vi hJcb`cZh\Y`dfYgYbhUhJcb`hcd]Wg"`
K YY_`&.`+`>i bY`!`%% >i bY	@YWifY	H\Y`W&bWYdhcZh\Y`±bXc!DUWJZJW'Xi ZZYfYbh UddfcUW\Yg`hc`W&bWYdhi U`]n]b[`U`fY[]cb/Wf]hJWU` [Ycdc`]hJWg'j g"`WUgg]WU``[Ycdc`]hJWg"
	Hi hcfJU`	
1		

	DUWJZJWgʻdck Yfʻdcʻ]hJWg3ʻK \Uhʻ]gʻh\Yʻ]a dUWrcZ

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

5```fY`Yj Ubh'Wci fgY`]bZcfa Uh]cb'UbX'fYei]fYX'fYUX]b[g'UfY'Uj U]`UV`Y'cb'h\Y'A ccX`Y'g]hY'Zcf'h\]g'Wci fgY'' D`YUgY'fY[i `Uf`m'W\YW_Zcf'i dXUhYgž'Ubbci bWYa Ybhgž'UbX'ch\Yf'fY`Yj Ubh'Wci fgY'fY`UhYX']bZcfa Uh]cb'cb A ccX`Y''

FYWca a YbXYX FYgci fWfg

 $\pm 0 \text{Zefa Uhjcb'UVci h'UXX]hjcbU``fYUX]b[g'UbX'fYgci fWfg']g'U`gc'a UXY'Uj U]`UV`Y'cb'h\Y'A ccX`Y'g]hY'Zef'h\]g Wci fgY"$

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUhjj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY