

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
8f [.] 5bh\cbm7\a]Y`	Ubh\cbm'W\a]Y`4 i bgk "YXi "Ui	,	K 65%*ž	
			FcVYfh	
			K YVghYf	
			6i]`X]b[

GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY.'f\$&Ł'-',)'(,)*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubg\cong h\Y'6YXY[U'`fPYbg]b[tcb'WJadig\cong ixc']Ubg\cong fh\'0'\fPYbg]b[tcb'WJadig\cong ixc']Ub\cong fh\'0'\fPYbJL'cZh\Y'`UbXg'k\YfY'YUW\cong ixc']bUk\Cong ixc']Ub'\cong fh\'0'\fPYB[ibbUk']Ub'\cong ixc']$$ $$CWJhYX''$$$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`Ug_g`ei Ygh]cbg`UVci ha i g]WWc[b]h]cb`UbX`VY\Uj]ci f.`<ck `Xc`k Y`dYfWr]j Y`a i g]WB`K \m\Wb Ub`]bghfi a Ybhgci bX`a i g]WB`K \xc`k Y`fYgdcbX`hc`UbX`dfcWfgg`h\Y`gci bX3`K \Uh\UddYbg`k \Yb`k Y UfY`dfYdUf]b[`UbX`dYfZcfa]b[`a i g]WB`K \m\Wb`dYfZcfa Yfg`VYWca Y`XYV]`]hUh]b[`mbYfj ci g3`K \Uh]g`h\Y fY`Uh]cb`VYhk YYb`a i g]WbgYl`UbX`W\YYgYWU_Y3`7 Ub`a i g]Wa U_Y`mci `ga UfhYfž'UbX`k \Uh]g`h\Y`A cnUfh 9ZZYWB`K \mXcYg`h\Uhhi bY`[Yhgh W_]b`a m\YUX3`<ck `c`X`a i ghU'dYfgcb`VY`VYZcfY`h\YmWbb`\YUf`UbX UddfYW]UhY`a i g]WB`Nci `k]```UXXfYgg`h\YgY`UbX`fY`UhYX`ei Ygh]cbg`h\fci [\`YI Ua]bUh]cb`cZh\Y`a i g]W dgmW\c`c[m`]hYfUh fYžUbX`h\Y`Wf]h]WU``Yj U`i Uh]cb`cZmci f`ck b`a i g]WU``Uhhjh XYgž`VY`]YZg`UbX`VY\Uj]ci fg"

7ci fgY @YUfb]b['Ci hWca Yg

- %" 91 d`U]b`\ck 'a i g]W]g`XYZ|bYX`Zfca 'U'dgm\%c`c[]WU`'dc]bhcZj]Yk "
- &" ⇒XYbhjZm\ck 'a i g]WdgmWkc`c[]ghg`h\]b_`UVci hUbX`UddfcUW\`ei Yghjcbg`WcbWYfb]b['a i g]W dYfWYdhjcb`UbX`Wc[b]hjcb"
- ' " 9a d`cmg_]``g`bYYXYX`hc`Z]bXž`UggYgg`UbX`XYg][b`a i g]WfYgYUfW\"

HYUWX]b[GhfUhY[]Yg

H\Y'Wci fgY'k]``VY'dfYgYbhYX'h\fci [\ZJVW!rc!ZJVW'`YWf fYg"'Hi rcf]U`g'UfY`\Y`X'cb`]bYž'k\]W\a YUbg'mci g\ci 'X'YI dYWfic 'XYj chY'i d'rc'cbY'\ci f'dYf'k YY_'fbch]bWi X]b['dfYdUfUh]cb'UbX'UggYgga Ybh'h]a YE'rc cb`]bY']bX]j]Xi U``UWfj]hJYg'UbX'Wc``UVcfUhj Y'UWfj]hJYg'k]h\ch\Yf'ghi XYbhg"'C h\Yf'UggYgga Ybh'hUg_g'k]``VY Wca d`YhYX']bXYdYbXYbhmi]b'UXX]hJcb'rc'h\Y'cb`]bY'hi rcf]U``hUg_g'rc'YbWci fU[Y'X]j YfgY'a cXYg'cZ`YUfb]b[']b UXXJhJcb'rc'hUddfcUWXYg"

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
HYgh	(\$1	Bch5dd`]WUV`Y	% <u>ž&</u> Ž
Ei]n'	%\$I	Bch5dd`]WUV`Y	%ž&Ž
Dcfh7c`]c) \$1	Bch5dd`]WUV`Y	&ž

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% HYgh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

 $\label{thm:linear} H\Y'HYghUggYggYg`YWi fY'WcbhYbhbchUggYggYX`]b`h\Y'Ei]n''`EiYgh]cbg`k]```dfcVY'VUg]WWcbWYdhg dfYgYbhYX`]b`h\Y'WcbfYbhg''``H\Y'hYghk]```Wcbg]ghcZai`h]d`Y'W.c]WY'UbX`g\cfhUbgk Yf`]hYag''`$

: YYXVUW_k]``Wcbg]ghcZU'bi a Yf]WU`a Uf_"`

5XX]h]cbU`XYhU]`g.

GYY'AccX`Y'Zcf'Zfh\Yf'XYhU]`

5ggYgga Ybh& Ei]n

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

: YYXVUW_`k]```Wcbg]ghcZU'bi a Yf]WU``a Uf_"`

5XX]h]cbU`XYhJ]`g.

 $\mathsf{G}\mathsf{Y}\mathsf{Y}^\mathsf{T}\mathsf{A}\;\mathsf{c}\mathsf{c}\mathsf{X}^\mathsf{T}\mathsf{Y}^\mathsf{T}\mathsf{Z}\mathsf{c}\mathsf{f}^\mathsf{T}\mathsf{Z}\;\mathsf{f}\mathsf{f}\mathsf{h}\mathsf{Y}\mathsf{f}^\mathsf{T}\mathsf{X}\mathsf{Y}\mathsf{h}\mathsf{U}]^\mathsf{T}$

5ggYgga Ybh' . 'DcfhZc`]c

GhUfhXUhY. Bch5dd`]WUV`Y

8 YW] g.

@|hYfUhi fY`fYj]Yk #gi a a UfmYl YfW]gYg/7 fYUh]j Y`Yl YfW]gY/9l dYf]a YbhU``a Yh\cX`Yl YfW]gY/DfYgYbhUh]cb 9l YfW]gY

&\$\$\$ k cfXg

 $A\ Uf_g`k\]```VY`bi\ a\ Yf]WU`ž`k\]h\`k\ f]hhYb`ZYYXVUW_`dfcj\]XYX''$

5XX]h]cbU`XYhJ]`g.

 $\mathsf{G} \mathsf{Y} \mathsf{Y}^\mathsf{T} \mathsf{A} \; \mathsf{C} \mathsf{C} \mathsf{X}^\mathsf{T} \mathsf{Y}^\mathsf{T} \mathsf{Z} \mathsf{C} \mathsf{f}^\mathsf{T} \mathsf{Z} \; \mathsf{f} \mathsf{f} \mathsf{h} \mathsf{Y} \mathsf{f}^\mathsf{T} \mathsf{X} \mathsf{Y} \mathsf{h} \mathsf{U}]^\mathsf{T}$

F Ygci fWYg DfYgWf]VYXYXY c'i fWYg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

5 WUXYa]W±bZcfa Uh]cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYi]Yk cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU`Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]miUbX'X]qUV]`]m/
- UbX'ch\Yf'YggYbhJU`UWUXYa]W]bZcfa UhJcbžgYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

.

⇒a U[Y'7fYX]h

D\chc'Vm? UVcca d]Wg'"Wca 'Zfca 'DYI Y`g

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{b}ck} YX[Y'hY'6YXY[U''dYcd'Y'k \c'UfY'hY'ffUX]h]cbU''W ghcX]Ubg'cZhY''UbXg'cb'k \]WX'I_BGK_{Ybg]b[hcb'WJa_di_q']q''cWJhYX'''$