

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

### GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
9X[Uf'; ÏaYn'7fin	Y"[ca YnWfi n4 i bgk "YXi "Ui		&'%; FcVYfh KYVghYf 6i]`X]b[	

#### GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&Ł'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong ``m5W_bck``YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJa di gŁż'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7Ua di gYgŁ'UbX'thY'B[i bbUk U` dYcd'Y'fbi gtfU']Ub'8YZYbWY': cfWY'5WJXYa m']b'7UbVYffUŁ'cZthY'`UbXg'k \YfY'YUW\cong WJa di g'cZl BGK ']g `cWJtYX''$ 

# 5ggYgga Ybh

### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
±b!7`Ugg'Dfc'YWfi	&\$ı	\$&#%\$#&\$&\$`%%) - `DA</td><td>%</td></tr><tr><td>G\cfh'9ggUm</td><td>' \$ı</td><td>&' #%\$#&\$&\$`%%) - `DA</td><td>%ž&</td></tr><tr><td>A U'cf'Dfc YWh</td><td>) \$1</td><td>&\$#%#&\$&\$`%%) - `DA</td><td>%ž&Ž</td></tr></tbody></table>	

### 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% ±0!7`Ugg Dfc YWh

GhUfh'XUhY.

8YHJ]`g.

 $Cb[c]b[``WfYUh]j Y`dfc'YWffgi W.`Ug`U`V`c[`cf`gcW]U``a YX]U`W fUh]cb½"k \] w``]g`k cf_YX`cb`]b`WUgg`]b`ga U``[fci dg"$ 

 $Gh XYbhg k ] ``fYWY] j Y j YfVU ``ZYYXVUW_ i] b `WUgg `UbX k f]hhYb `ZYYXVUW_ ij ]U `A ccX`Y/gcW]U ``a YX]U `ghUh]gh]Wg UbX`gcW]U ``a YX]U `ZYYXVUW_ k ] `` ]a dUWih\Y [ fUXY"$ 

#### 5XX]h]cbU`XYhU]`g.

7\YW\_AccXYZcfacfY]bZcfaUh]cb

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh& G\cfh9ggUm

GhUfh'XUhY.

8 YHJ] g.

%\$\$\$ k cfXg"

Ghi XYbhg k ]``fYWY]j Y k f]hhYb ZYYXVUW\_ij ]U Hi fb]h]b"

#### 5XX]h]cbU`XYhU]`g.

7\YW\_'AccX`Y'Zcf'acfY']bZcfaUh]cb

5ggYgga Ybh' . 'A U'cf'Dfc YWh

GhUfh'XUhY.

#### 8YHJ]`g.

&) \$\$`k cfXg/cfž`U`a YX]U`Zcfa Uh`fU`j ]XYcž`U`V`c[ž`U`dcXWUghž`YhWłż`k \]W\`bYYXg`hc`VY`Z|fgh`Uddfcj YX`Vmh\Y Wcbj Ybcf"

H\]g`]g`h\Y`Z]bU``UggYgga Ybh'hUg\_"

Ghi XYbhg k ]``fYWY]j Y k f]hhYb ZYYXVUW\_j jU'Hi fb]h]b"

#### 5XX]h]cbU`XYHJ]`g.

7\YW\_'AccX`Y'Zcf'acfY']bZcfaUh]cb

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ $H \ fb \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \en$ 

# 5 https://bxubwww.Fyei ]fya Ybhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj ]Yk '`YWhi fY`fYWcfX]b[g"

# 7ci fgY'GWXYXi `Y

### J]Yk 'WUgg'ha YhUV'Y

# H]a YtUV`Y

8UhY	Hmd Y	7 cbhYbh	
K YY_'% '%( 'GYdhYa VYf'!	@YWnifY	K Y`Wca Y'hc'h\Y'Yj YfmXUm`]ZY']b'h\Y'dcghX][]hU`'YfU'	
% 'GYdhYa VYf		? Ymk cfXg. '8][]h]gUh]cbž'hYVXbc`c[mž'Y;YfmXUm`]ZYž	
		W'h fyfgł	
K YY_`&.`&%`GYdhYa VYf`!	@YWhifY	bXYfgHJbX]b[ 'U`[ cf]h\a ]WUbX`XUHJ`W`hi fYg	
&) 'GYdhYa VYf			
		? Ymk cfXg. 5`[ cf]h\a gž'W`hi fYžXUhJZJWJh]cbž Ui hca Uh]cb	
K YY_''.'&, 'GYdhYa VYf'! &'C WcVYf	@YWnifY	l bXYfgHJbX]b[ 'X][ ]HJ``dUffjMjdUfjcbžWcaai b]fjYg UbX`]XYbfjfjYg	
		? Ymk cfXg. 7 cbgi a dh]cbždfcXi Wh]cbždUfh]W]dUh]cbž Wca a i b]h]Yg	
	5ggYgga Ybh	±0!WUgg dfc YWifHi fb]hjb gi Va ]gg]cbŁ	
K YY_'(.') 'C WcVYf'!'- C WcVYf	@YWhi fY	I bXYfgHJbX]b[ 'Df]j UWmUbX'gi fj Y]``UbW	
		? Ymk cfXg. Df]j UWmžgi fj Y]``UbWřžWcbgi a dh]cbž gcW]U``a YX]U	
K YY_').'%&'CWtcVYf'!'%* CWtcVYf	F YUX]b[	I bXYfgHJbX]b[ 'A cV]`]hmUbX'k cf_']b'h\Y'U`[ cf]h\a ]W U[ Y	
		? Ymk cfXg. A cV]`]hmžk cf_žUi hca Uh]cbž[][ 'YWcbca m	
K YY_'*.'%-'CWcVYf'!'&'	FYUX]b[		
C WrcVYf	5ggYgga Ybh	G\cfhYggUmfHi fb]h]b gi Va ]gg]cbŁ	
K YY_'+.'&*'C WcVYf'!'' \$ C WcVYf	@YWhi fY	l bXYfgHJbX]b[ 'dc`]hJWJ`X][ ]HJ`W`hi fYg	
		? Ymk cfXg. 'Dc`]h]WgžgcW]U``a cj Ya YbhgžfYg]ghUbW/g	
K YY_', .`&Bcj Ya VYf'!'* Bcj Ya VYf	@YWh fY	I bXYfgHJbX]b[ '`Y]gi fY'W' hi fYg	
5, 14 111		? Ymk cfXg.; Ua ]b[ ž'V]b[ Y'k UhW ]b[ ž'gcW]U`a YX]Už WY`YVf]h]Yg	
K YY_'`- 'Bcj Ya VYf'! % 'Bcj Ya VYf	@YWn fY	I bXYfglUbX]b[ 'X][ ]lU``]bYei U`]lfJYg'UbX'U`[ cf]l\a ]W j i `bYfUV]`]lm	
		? Ymk cfXg. `]bYei U`]h]Ygž'gi fj Y]``UbWY`WUd]hU`]ga ž X][ ]hU``f][ \hg	
K YY_"%\$."%*"Bcj Ya VYf"! &\$"Bcj Ya VYf	@YWhifY	I bXYfglUbX]b[ 'X][ ]lU``Z hi fYg	

1		? Ymk cfXg.: i hi fYžfcVchgžXUhUžfcVchgžUi hca Uhjcb		
	5ggYgga Ybh	A Ućf Dfc YWifHi fb]h]b Gi Va ]gg]cbŁ		

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhjb['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bz'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=\mathred{Image: Jmci UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbz'Vi hmci a i ghfYhJ]b'mci f'h]\W\_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\WJh]cb"'\=\mathrea{Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k]'`
Ui hca Uh]\WJ'`mfYW']j Y'U'h]\W\_Yhbi a VYfzVi h]Zmci 'hY'Yd\cbY'mci 'k]'`bYYX'hc'gdY\WZ\WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUhjj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

