

MDIA5022

7 cfdcfUhY 'UbX' ⇒bhYfdYfgcbU`'7 ca a i b]WUh]cb

HYfa 'H\fYY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
A]WXUY`?Ybh	n') &(\$-) 4 i bgk "YXi "Ui			

GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY. f\$&Ł'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

DfcZ/gg]cbU`cf[Ub]nUh]cbU``Waaaib]Whhcb']g'ibXYfd]bbYX`Vmh\Y`UV]]mhcZcfa'dcg]hj Y`fY`Uh]cbq\]dg
UbX`bUj][UhY`X]ZYfYbWg']b'h\Y`k cf_d`UWW''K \]`Y`k Y`cZhYb'UggiaY'h\UhYi g]bYgg'Waaib]yYgj'Waaib]g'U
fUh]cbU``dfcWggZk cf_d`UW\g'UfY`U`gc']adUWYX`cb'VmYach]cbg'h\UhYad`cmYYg'WUffmk]h\`h\Ya'''K\]Y
]ggi Yg`cZ`Ub[i U[YžW`hifY'UbX`Yach]cbi' bXYfd]b'U```]bhYfdYfgcbU``Waaib]Wh]cbz'ghi XYbhg']b'h\]g
WaifgY`UfY`dUfh]W``Uf`mYbWaifU[YX`rc'Udd`mh\Ycf]Ygz'WabWdhg'UbX`]XYUg'cZ]bhYfdYfgcbU``Waaib]Wh]cb
hc']ggi Ygz'W\U``Yb[Ygz'UbX'cddcfhib]h]Yg']b'WafdcfUhY'UbX'ch\Yf'cf[Ub]nUh]cbU``WabhYlhg''H\Y'WaifgY'k]`
ZcW g'cb'Udd`]WUh]cb'cZ]bhYfdYfgcbU``Waaib]Wajb'b'Wahg'UbX'dfcWygyg'hc'cf[Ub]nUh]cbU``WabhYlhgz
cddcfhib]h]Ygz'W\U``Yb[Yg'UbX`]ggi Yg'''? Ymhcd]Wg']b'h\]g'WaifgY']bWiXY'cf[Ub]nUh]cbU``Waaib]Wh]cb'UbX
X]gWaifgYzbY[ch]Uh]cbz'dYfgi Ug]cbz'UXj cWUWnzaUbU[]b['W\Ub[Yzj U'i]b['X]j Yfg]hmzYh\]WO``XYWJg]cb!
aU]b['UbX'WabZ]WhfYgc`i h]cb''

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 5dd`m`]bhYfdYfgcbU``Wcaaib]WUh]cb`WcbWYdhg`UbX`dfcWYggYg`hc`cf[Ub]gUh]cbU``WcbhYl hg`UbX dfcV`Yag
- &" 8Ya cbgffUhY _bck `YX[Y`cZcf[Ub]qUhcbU``Wca a i b]\WUhcb `Wcb\Wdhq
- ' " 5dd`mbY[ch]Uh]cb'UbX'dYfgi Ug]cb'g_]``gž'UbX'h\Ycf]Yg'cZa UbU[]b['W\Ub[Y
- ("91Ua]bY'h\Y']adcfrUbW'cZjU'i]b['X]jYfg]mžYh\]WU`XYWJg]cb'aU_]b['UbX'WcbZ]WifYgc'ih]cb

HYUWX]b['GhfUhY[]Yq'

HYUW]b[GhfUhY[]Yg.

7 cfdcfUhY 'UbX' +bhYfdYfgcbU` '7 ca a i b]\Wh]cb']g'XYg][bYX' hc'YbWci fU[Y'UbX'WXU``Yb[Y'mci 'hc'fYZYWhcbUbX']a dfcjY'mci f'ck b'Wca a i b]\Wh]cb'dfU\Wh]Wh"

H\Y`Wci fgY`]g`XY`]j YfYX`]b`U`gYa]bUf`Zcfa ž`k \YfY`X]gW gg]cbgž`UbU`mg]g`UbX`Udd`]WUf]cb`cZ_YmWcbW'dfg UbX`fYW'bhfYgYUfW\Z]bX]b[g`UfY`i gYX`fc`XYj Y`cd`U`XYYdYf`i bXYfgfUbX]b[`cZgffUfY[]WWca a i b]WUf]cb" H\Y`gi V^YWfi]g`XYg][bYX`Ufci bX`U`a]l`cZUWfj]fjYg`fc`Vi]`X`dUfffJWJdUbfgfiJbfYfYgfiUbX``YUfb]b["`H\Y`fk c\ci f`WUgg`dYf]cX`YUW\k YY_k]``]bWi XY.

- U'gYWjcb'cb'_Ymdf]bWjd'Ygžk\]W\'k]``]blfcXi W\'h\Y'fY'Yj UbhWcbW\'dlg'UbX'h\YcfmcZWcfdcfUlY UbX']bhYfdYfgcbU`Wca a i b]WUljcb"
- U'gYf]Yg'cZX]gW gg]cbg'UbX'UWfj]h]Yg'U]a YX'UhUdd'm]b['h\Y' Ymdf]bWd'Yg"

Hc cVHJ]b h Y Z ``VYbYZ]hg Zca h]g Wci fgYzgh XYbhg UfY YbWci fU[YX hc YI hYbX mci fgY`ZVYmcbX h Y]f Wca ZcfnncbY"'H\Una YUbg UWlj Y`mdUff]W]dUl]b[i]b h Y Wcbj YfgUl]cbg UbX X]gW gg]cbg i]b WUgg Ug Vch 'U `]ghYbYf'UbX gdYU_Yfz VY]b[cdYb hc UbX k]``]b[hc WcbhYa d`UhY U fUb[Y cZdYfgdYWlj Yg UbX hfmbYk k Umg cZWca a i b]WUl]b[zfYZYWl]b[XYYd`mcb h Y]f ck b Wca a i b]WUl]cb ghfYb[h g UbX k YU_bYggYgz UbX VY]b[k]``]b[hc []j Y Wcbghi Wlj Y ZYYXVUW_hc ch Yfg]b cfXYf ZcghYf dcg]h]j Y UbX YZYWlj Y Wca a i b]WUl]cbz dUff]W `Uf m]b X]j YfgYz hYUa !VUgYXz k cf_!fY`UhYX WcbhYl hg" K Y \cdY h Uhgh XYbhg k]```ZYY`h Uhh]g Wci fgY cZYfg U gUZY Ybj]fcba Ybhhc hmci hbYk 'ghmYg cZWca a i b]WUl]cb k \]WX h\Yma Um`UhYf Udd`m]b h\Yk cf 'd`UWY"

Ghi XYbhgʻUfYʻUg_YXʻhcʻfYZYWhicbʻh\YʻgYhfYUX]b[gʻVYZcfYʻYUWXʻWUggžʻUbXʻVf]b[ʻh\Y]fʻei Ygh]cbq UVci hh\YʻfYUX]b[gʻVUgYXʻcbʻh\Y]fʻck bʻi b]ei YʻYI dYf]YbWYgʻhcʻYUWXʻWUgg"	gʻUbXʻ]XYUg

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
±b!7`Ugg'9l Ua '	' \$1	Bch5dd`]WUV`Y	%ž&Ž Ž(
; fci d'dfYgYbfUf]cb'UbX fYdcfhi	' \$ı	Bch5dd`]WUV`Y	%ž&
F YgYUfWX 'DUdYf'	(\$1	Bch5dd`]WUV`Y	%ž&ž ž(

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% ±17`Ugg 9l Ua

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

9ggUmlVUgYX`]b!WUgg`Yl Ua "`E i Ygh]cbg`k]```VY`dfcj]XYX`U\YUX`cZh]a Y/ei Ygh]cbg`k \]W\`gh XYbhg`k]`` Ubgk Yf`]b`h\Y`Yl Ua `k]```VY`fUbXca `mgY`YWhYX`cb`h\Y`XUmcZh\Y`Yl Ua "

K f]hhYb ZYYXVUW_dfcj]XYX cb Yl Ua gžk \]WX k]``VY fYh fbYX hc gh XYbhg"

5XX]h]cbU`XYhU]`g.

H\Y'YI Ua 'k]``'VY'U'ZUWY!hc!ZUWY'YI Ua ž'[]j Yb'j]U'Ncca ž'YhW'

@Yb[h\.'&ž)\$\$Ž'kcfXg

8YHJ]`g.

&) \$\$`k cfXg`Zcf`k f]hhYb`fYZYWh]cb"`H\]g`]g`h\Y`Z]bU``UggYgga YbhhUg_"

K f]hh/b ZYYXVUW_dfcj]XYX j]U Hi fb]h]b"

 $\label{thm:continuous} \mbox{Hi fb]h]b$'gYhh]b$[.$'H\]g$'Ugg][ba Ybh]g$'gi Va]hhYX'h\fci [\'Hi fb]h]b$'UbX'gh XYbhg`WUb'gYY'Hi fb]h]b$'g]a]`Uf]hm fYdcfhg"$

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

GYY gm`UVi g" K]` VY dfcj]XYX"

FYWca a YbXYX FYgci fWYg

BchUj U]`UV`Y

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

H\fci [\'h\Y'a UbXUh\X'\Zcfa i `U'h\Uh'U`fYUXmYI]ghg"`5bX'j]U']b\Zcfa U``Wcbj YfgUh]cbg"`

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

-Zmci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\'Hi fb]h]bž'd`YUgY hY`Yd\cbY'9I hYfbU``Gi ddcfhcb'-',)'''' %cf'Ya U]`'h\Ya 'cb'YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@-69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.#gi VYWn[i]XYg"`]VfUfm"i bgk "YXi "Ui #\`]gY#UVci hY`]gY