

## 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## GHJZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
HUb1J:8fY\Yf	h'XfY\Yf4 i bgk "YXi "Ui	,	FcVYfh KYVghYfž@YjY` 'žFcca'' ‰G	

#### GWXcc~7cbHJWh~\pdzcfa Uh]cb

GWVcc``cZfVY`5flg`UbX`AYX]U

Fcca " &ž'@Yj Y`" ž'FcVYfhK YVghYf'6i ]`X]b[ fl %(Ł

D\cbY.'f\$&Ł'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

gh/f<sup>-</sup>6i]<sup>X</sup>]b[<sup>-</sup>fl

## 7ci fgY 8YW] g

7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y 7ci fgY

.

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 8 YgWf]VY 'UbX'X]gW gg 'gca Y 'cZh\Y 'WYblfU``XYVUhYg 'fY`Uh]b[ 'hc 'a YX]U 'Yh\]Wgz'gcWJU``^ gh]WY 'UbX \i a Ub 'f][ \hg
- &" 5dd`m'WcbW'dhg`W'bhfU``hc`a YX]U`Yh\]Wg`UbX`\i a Ub`f][\hg`UbX`Wf]h]WU``m'UbU`mgY`h\Y`Yh\]WU``UbX dc`]h]WU`]a d`]WUh]cbg`cZa YX]U`dfUWf]W'g`UbX`fYdfYgYbhUh]cbg"
- '" 7 fuZhu WY Uf Ub X dY fgi Ug]j Y k f]hh b Uf[i a Ybh

## HYUW ]b[ 'GhfUhY[ ]Yg'

F YUX]b[gžj]XYcg'UbX``YWi fYg'k]```dfcj]XY'\]ghcf]WU``WcbhYI hUbX'\_YmWcbW'dhg'Zcf'i bXYfghUbX]b[fY`Uh]cbg'VYhk YYb'a YX]Už'Yh\]WgžgcW]U``^ gh]WY'UbX`\i a Ub'f][\hg''@YUfb]b['UW]j ]h]Yg'k]```gi ddcfhighi XYbh `YX'Yb[U[Ya Ybhk]h\`]ggi Yg'UbX'WJgY'ghi X]Yg''H\]g'k]````YUX'hck UfXg'h\Y'Z]bU``Ugg][ba Ybhž'U'a U'cf YggUmž'h\Uh'UbU`mgYg'U'W'ffYbh]ggi Y']b'a YX]Už'gcW]U``^ gh]WYžYh\]Wg'UbX'\i a Ub'f][\hg''

.

## 5ggYgga Ybh

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'tUg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ CihWca Ygʻ5ggYggYX
7 ca a YbHJfmcb'U'k YY_`m hcd]W	' \$I	( 'g\cfhWca a YbfUf]Yg'cb' ( 'cZ h\Y'k YY_`mhcd]Wg'Xi f]b[ K YY_g'&'!'-	%ž &
Dcgh/f <sup>-</sup>	&\$ı	%'dcghYf'cb'%'cZh\Y'kYY_m hcd]Wg'Xif]b['KYY_g''!'-	%ž &
A U⁄cf <sup>·</sup> 9ggUmi	) \$1	\$-#\$,#&\$&%'%\$.\$\$'DA	%ž <sup>*</sup> &

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% 7ca a YbhUfmcb U'k YY\_`mhcd]W

GhJfhXUhY. ( 'g\cfh'Wca a YbhJf]Yg'cb ( 'cZh\Y'k YY\_`mhcd]Wg'Xi f]b[ 'K YY\_g'' !! '%\$

@Yb[h\.(I)\$\$!kcfXWcaaYbhUf]Yg

8 YW] g.

'\$!dYfWybh

( 'I ') \$\$!k cfX 'Wca a YbhUf]Yg''

Gh XYbhg k ]``dfcXi W g\cfhWa a YbhJf]Yg ]b fYgdcbgY hc dcghYfg dfYdUfYX Vmh\Y]f dYYfg"

: YYXVUW\_'j ]U'h\Y'Wti fgY'@A G"

#### 5XX]h]cbU`XYhJ]`g.

: i ff\\Yf'XYfU]`g'cb'f\\]g'Ugg][ ba Ybfik ]``'VY'Uj U]`UV'Y']b'A ccX'Y

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhh]b[."H\]g"ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"uf]hmfYdcfhg" \\ \end{tabular}$ 

5ggYgga Ybh& DcghYf

GhUfhiXuhy. %dcghyficb %cZh\Yik YY\_mhcd]WgiXi f]b[iK YY\_gii !i%

@Yb[h\. 5' Dcgh/f

8 YHJ] g.

&\$!dYfWbh

Ghi XYbhgʻk]``dfcXi WYʻUʻdcghYfʻcbʻcbYʻcZh\Y`kYY\_`mhcd]WgʻfY`Uh]b[ˈhcˈa YX]UʻYh\]WgʻžgcWJU``1 gh]WY`UbX\i a Ubʻf][\hg"

: YYXVUW\_ij ]U'h\Y'Wti fgY'@A G"'

#### 5XX]h]cbU`XYhJ]`g.

: i ft\Yf'XYfU]`g'cb't\]g'Ugg][ ba Ybhk ]``VY'Uj U]`UV'Y']b'A ccX'Y

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

5ggYgga Ybh' . A U'cf 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.~&\$\$\$!~&) \$\$\*k cfXg

8YHJ]`g.

- ) \$!dYfW\bh
- &) \$\$ k cfXg"

: YYXVUW\_ij ]U'h\Y'Wci fgY'@A G"

.

#### 5XX]h]cbU`XYhU]`g.

: i ff\Yf`XYfU]`g`cb`f\]g`Ugg][ ba Ybhk ]``VY`Uj U]`UV`Y`]b`A ccX`Y

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$ 

## FYgci fWyg

#### DfYgWf]VYX F Ygci fWYg

DfYgWf]VYX`fYgci fWfg`k]```VY`a UXY`Uj U]`UV`Y`j ]U`A ccX`Y"`DfYgWf]VYX`fYgci fWfg`k]```]bWi XY`gWkc`Uf`m`]hYfUhi fYž`Yl dYfhWca a YbhUf]Ygž`dcXWUghg`UbX`j ]XYcg"

#### FYWca a YbXYX FYgci fWyg

F YWca a YbXYX'fYgci fWrg'k ]```VY'a UXY'Uj U]`UV`Y'j ]U'A ccX`Y"'F YWca a YbXYX'fYgci fWrg'k ]```]bWi XY gW.c`Uf`m`]hYfUh fYžYl dYfhWca a YbHUf]YgždcXWUghg'UbX'j ]XYcg"'

#### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\Y'5FHG&\$-\*'hYUW\]b['hYUa'j U'i Yg'ghi XYbhZYXVUW\_"'Ghi XYbhZYXVUW\_']g'k Y`Wca Y`UhUbmhJa Y"
: cfa U`'ZYYXVUW\_'a YW\Ub]ga g']bWi X]b['h\Y'a m9I dYf]YbW''gi fj Ymk]``'VY'Uj U]`UV`Y"'+bZcfa U`'ZYYXVUW\_
k |``'VY'YbWci fU[YX'h\fci [\ci hh\Y'HYfa "'

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hy'Yd\cby'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

# 5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['lc.